Chapter 6 Counting.

6.1 The basics of counting

To count all the ways something can happen you could just try to list all the ways.

Example ① How many ways can you get dressed if you're choosing between three shirts A, B, C and two pants P, Q?

Solution: List = AP, BP, CP, AQ, BQ, CQ so there are six ways to get dressed.

This example shows an important principle.

> The Product Rule. A procedure requires two tasks. If the first task can be done in X ways and the second in Y ways, then there are XY ways to do the procedure.

Here, the second task may depend on the first but there are always Y ways to do the second. If there was a third task required, that could always be done in Z ways, then there would be XYZ ways to do the procedure and so on for more tasks.

Example ② You see an online product that comes in 6 colors and 7 sizes. How many possibilities is that?

Solution: We can choose the color in 6 ways and for each color there are 7 size options. By the product rule this makes 42 possibilities.

Example ③ Let $S = \{1, 2, \dots, 6\}$ and $T = \{3, 4, \dots, 9\}$. Find $|S \times T|$.

Solution: Remember from section 2.1 that

$$\boxed{S \times T = \{(a,b) \mid a \in S,\ b \in T\},}$$

with (a,b) an ordered pair. There are 6 choices of the first element a from S and 7 choices of the second element b from T:

$$S \times T = \{(1,3), (2,3), \dots, (6,9)\}$$

and $|S \times T| = 42$.

- See examples 1, 2, 3 on p 386.

Example ④ Suppose a multiple choice test has 5 questions, each with 4 possible answers. If a student just randomly chooses answers, how many ways can this be done?

Solution: 20 ways? <u>No</u>. This time there are really 5 tasks (each question) with 4 ways to answer each one.

So $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 4^5 = 1024$ ways to randomly answer the test.

<u>Example</u> 5 How many bitstrings are there of length 6?

Solution: This is a product rule application with six tasks. There are 2 ways to choose the first bit 0 or 1. 2 ways for the second ... 2 ways for the sixth.

So $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^6 = 64$ possible bit strings of length 6.

<u>Example</u> 6 Let $S = \{-1, 0, 1, 2, 3, 4\}$. How many subsets does S have?

Solution: This is also a product rule application with six tasks. In the first task we decide if -1 will be in the subset – two options yes or no. Then decide if 0 is in or not ... decide if 4 is in or not.

So $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^6 = 64$ possible subsets of S.

We saw the correspondance between subsets and bit strings already in section 2.2.

$\{-1,0,1,2,3,4\}$ ---- 111111

$\{-1,0,1,2,3\}$ ---- 111110

$\vdots$ $\vdots$

$\{-1, 1, 2\}$ --- 101100

$\vdots$ $\vdots$

$\{\}$ --- 000000

- More examples 5-9 p 387.

Example (7) Suppose you have room for one more class in your schedule for next semester. You see 4 possible math classes and 3 possible computer science classes. How many options do you have?

Solution: Now there is really only one task - choosing one class. Altogether there are $4+3 = 7$ options.

The Sum Rule. If a task can be done in X ways or Y other ways (all these ways different) then there are $X + Y$ ways to do the task.

If the task can also be done in Z other ways then the total number of ways would be $X+Y+Z$ and so on.

Example ⑧ You are looking to buy a new car and narrow your search to 4 Fords, 3 Toyotas and 5 Hondas. How many ways can you buy a new car?

Solution: By the sum rule its $4+3+5 = 12$ ways you can buy a new car.

- See examples 12-14 p 389.

In terms of sets,

Product rule: $|A \times B| = |A||B|$

Sum rule: $|A \cup B| = |A| + |B|$ if no elements in common