

## 1.2 Applications of propositional logic

### Translating English sentences

We can make sentences logically precise by writing them in propositional logic.

Example (1) Translate this into a logical expression

"You can board the plane if you have a ticket and valid passport."

Solution: Look for the words "and", "or", "not" and "if" or words that have the same meaning. The sentence can be broken into these propositions

$p$  says "you can board the plane"

$q$  says "you have a ticket"

$r$  says "you have a valid passport"

and we get  $(q \wedge r) \rightarrow p$

- See more examples on p 17.

Propositional logic can be used to check if a set of conditions is consistent.

This means it is possible to satisfy all the conditions - otherwise they are inconsistent.

Example (2) A celebrity is hiring a personal assistant with these conditions:

- You must speak 3 languages or be 30 years old or younger.
- You must not use Twitter or be over 30.
- If you use Twitter you must not speak 3 languages.

Are these conditions consistent?

Solution: Let  $p$  be the proposition that the applicant has age  $\leq 30$ ,  $q$  says they speak 3 languages and  $t$  says they use Twitter.

The conditions become

$$p \vee q, \quad \neg t \vee \neg p, \quad t \rightarrow \neg q$$

Can all three  $\uparrow$   $\nearrow$   $\rightarrow$  be true?

Suppose the applicant uses Twitter. Then  $t$  is T and  $\neg q$  must be T so  $q$  is F. If  $q$  is F then  $p$  must be T but then  $\neg t \vee \neg p$  is F.

So there is a problem if the applicant uses Twitter. If  $t$  is F then the conditions can be satisfied if  $p$  or  $q$  is T.

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## Boolean searches

If you type words into a search engine like Google then it automatically assumes you want to find pages containing all the words.

So searching for "george boole" is the same as "george" AND "boole".

If you use OR then Google says this is a way to combine two searches. It looks like it just uses AND though.

More useful is the NOT operator which you can write with a minus sign.

Searching "logic" comes back with Apple products and a rapper so try

"logic -apple -rapper"

which means "logic"  $\wedge$   $\neg$ "apple"  $\wedge$   $\neg$ "rapper".

to get good results on logic in computer science for example.

## Logic puzzles

If you enjoy puzzles then logic puzzles can be fun and challenging.

Famous examples are set on an island where people either always tell the truth (knights) or always lie (knaves). You can't tell them apart by looking - just using logic.

Example (3) You meet two people A and B on this island.

A says "we're both knights"

B says "A is a knave".

Are they knights or knaves?

Solution: Let's look at the possibilities.

Suppose A is a knight. Then he's telling the truth and they're both knights.

But this contradicts B saying he's a knave. Therefore A must be a knave.

So B told the truth and B is a knight.

Example (4) Is it possible for someone on the island to say "I am a knave"?

Solution: Not possible - can you see why?

- More examples p 19, 20.