1.1 Propositional Logic (continued)

We saw that propositions with labels like p, q, r can be either true (T) or false (F).

We can apply operations $\neg, \wedge, \vee, \oplus, \rightarrow$ to get new propositions

Example ① Evaluate $\neg p \vee (p \wedge q)$ when

p is T and q is F.

Solution: First note that negations should always be done first and $\neg p$ is F so that we get

$$F \vee (T \wedge F) = F \vee F = F.$$

So the compound proposition is false.

The implication $p \rightarrow q$ has three related statements

The contrapositive	$\neg q \rightarrow \neg p$
the converse	$q \rightarrow p$
the inverse	$\neg p \rightarrow \neg q$

If we compare $p \rightarrow q$ with its contrapositive

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$\neg q \rightarrow \neg p$
T	T	T
T	F	F
F	T	T
F	F	T

we see they have the same truth tables, so we call them equivalent.

So an implication and its contrapositive mean the same thing logically. They are not the same as the converse or inverse (not equivalent).

See example 9 on page 9.

Biconditional statement

If p and q are propositions then $p \leftrightarrow q$ is read "p if and only if q". This new proposition is true if p and q have the same truth value. It is false otherwise.

Remember "p only if q" means $p \rightarrow q$,
"p if q" means $q \rightarrow p$

So here we have both and $p \leftrightarrow q$ is equivalent to $(p \rightarrow q) \wedge (q \rightarrow p)$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Example ② Let p say "it is raining" and q say "the streets are wet". If q is true and $p \leftrightarrow q$ is true, what can we say about p?

Solution: p must be true – so it is raining.

Truth tables for compound propositions

We can understand combinations of propositions by writing every possible truth value in a table.

Example ③ Give the truth table for $p \wedge \neg p$.

Solution: There are only two possibilities for p so we need two rows. In the second column we find $\neg p$ and the third column gives the result we want:

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

Then $p \wedge \neg p$ is always false.

If we have a compound proposition with p and q then we need 4 rows. For p, q and r we need 8 rows

Example ④ Give the truth table for

$$(p \to q) \wedge (q \leftrightarrow r)$$

Solution:

p	q	r	$p \to q$	$q \leftrightarrow r$	$(p\to q)\wedge(q \leftrightarrow r)$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	F	F
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	F	F
F	F	F	T	T	T

See also example 11 on p. 10

Bit operations

Computers work with ones and zeros. One always represents True and zero represents False. So we can rewrite all our truth tables with 1s and 0s if we want – see table 9 on p. 12

A bit means something that is either 0 or 1 (binary digit) and a

bit string just means a sequence of bits.

So 001011101000 is an example of a bit string of length 12.

A common way to combine two bit strings of the same length is to use $\wedge$ on each pair of corresponding bits — this is called <u>bitwise And</u>.

For example, the bitwise And of 110, 010 is

$$\begin{array}{ccc} 1 & 1 & 0 \\ 0 & 1 & 0 \end{array} \qquad \text{answer } 010$$

compare and apply $\wedge$

Similary we have bitwise Or with $\vee$ and bitwise Xor with $\oplus$ (exclusive or).

<u>Example</u> 5 Let $A = 011010$ and $B = 110100$.

(a) Find the bitwise Xor of A and B. Then
(b) find the bitwise Xor of this answer with B.

Solution:

$$\begin{array}{llll} & & 011010 & A \\ & & 110100 & B \\ \hline (a) & \text{Xor} & 101110 & A \oplus B \end{array}$$

$$\begin{array}{llll} & & 101110 & A \oplus B \\ & & 110100 & B \\ \hline (b) & \text{Xor} & 011010 & \text{(back to } A\text{).} \end{array}$$