Chapter 1 Logic + proofs.

1.1 Propositional Logic

We are used to variables like x that can represent any number. Propositions are a different kind of variable that can take only two possible values: True or False.

Examples

① p says "$3+4=7$"

then p is a proposition and p is True.

② q says "the earth is flat"

then q is a False proposition.

③ r says "$x=2$"

here, r is not a proposition because we cannot say if it is true or false.

④ s says "Who are you?"

s is not a proposition.

⑤ $x = 12$

here x is not a proposition, it's a number.

⑥ t says "$1<2$ and $3<10$"

then t is a true proposition.

The rules of logic tell us how we can combine propositions — just like how the rules of arithmetic let us combine numbers.

Negation

If p is a proposition then $\neg p$ is read "not p". This new proposition takes the opposite truth value.

$\neg p$ says "it is not the case that p"

So if p says "I own a Ferrari" then

$\neg p$ says "it's not the case that I own a Ferrari"

In other words $\neg p$ says "I don't own a Ferrari".

Conjunction

If p and q are propositions then $p \wedge q$ is read "p and q". This new proposition is true if p and q are both true. Otherwise it is false.

Example: Suppose p says "$1 < 3$" and q says "$4 = 5$". Then $p \wedge q$ says "$1 < 3$ and $4 = 5$". We see that p is true, q is false and $p \wedge q$ is false.

Disjunction

If p and q are propositions then $p \vee q$ is read "p or q". This new proposition is false if p and q are both false. Otherwise it is true.

So $p \vee q$ is true if p or q is true (or both)

Inclusive or

Example: p says "$1 < 3$", q says "$4 = 5$" then $p \vee q$ is true

but $(\neg p) \vee q$ is false

Truth tables show every possibility when you combine propositions

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

And table

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Or table

Example: Suppose p is true, by looking at the tables is it possible for $p \wedge q$ or $p \vee q$ to be false?

Take a look...

More operations:

Exclusive Or

If p and q are propositions then $p \oplus q$ means exclusive or and is true exactly when p, q have different truth values.

Implication

If p, q are propositions then $p \to q$ is read "p implies q". It is always true except when p is true and q is false.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

The implication $p \to q$ is very important in logical reasoning and proofs. If $p \to q$ is true then the truth value of p tells you about the truth value of q, at least when p is true:

If $p \to q$ is true and p is true then q must be true.

We can say "if p then q" or "q if p".

Also if $p \to q$ is true and q is false then p must be false.

we say "p only if q" then also to mean $p \to q$.

Example: Let p be the proposition "I passed Precalulus" and let q say "I can take Calculus". Then the implication $p \to q$ means
"I passed Precalculus implies I can take calculus"
which is true. Equivalent ways to say it

- "If I passed Precalc then I can take Calculus"
- "I can take Calculus if I passed Precalc"
- "I passed Precalc only if I can take Calculus".

Example: Is this implication true
"the earth is flat implies $3+4=7$"?

Answer - Yes this is true, even though it seems strange. Implications are always true if the first proposition is false. So

"the earth is flat implies $3+4=10$" is also true.