Review of sections 2.1, 2.2, 2.3, 2.6, 3.1

2.1 Sets

Sets are collections of objects. For example $S = \{5, 6, 10, 13\}$ is a set containing the objects (elements) 5, 6, 10 and 13.

Notation

$\in$ is an element of
$\subseteq$ is a subset of
$\subset$ proper subset
ϕ empty set

Also $P(S)$ means the power set of S which is the set of all subsets.

For example

$$P(\{1,2\}) = \{\phi, \{1\}, \{2\}, \{1,2\}\}.$$

Also $|S|$ means the cardinality of S which is the number of its elements. So $|S| = 4$ here. Note that $|P(\{1,2\})| = 4$ as well.

We also looked at the Cartesian product of two sets A and B:

$$A \times B = \{(a,b) \mid a \in A,\ b \in B\}$$

ordered pairs.

set builder notation

2.2 Set operations

There are different ways to combine two sets A and B

$A \cup B$ union
$A \cap B$ intersection
$A - B$ difference
$A \oplus B$ symmetric difference

Example ① For $A = \{1,2,3\}$, $B = \{3,4\}$

find $A \cup B$, $B - A$, $A \oplus B$, $A \cap B$.

Solution:
$$A \cup B = \{1,2,3,4\}$$
$$B - A = \{4\}$$
$$A \oplus B = \{1,2,4\}$$
$$A \cap B = \{3\}.$$

Example ② For $A = \{1,2,3\}$, $B = \{3,4\}$, $C = \{1,4\}$

find $A \cap B \cap C$.

Solution: Can write this as $(A \cap B) \cap C$ (since intersection is associative)

and $A \cap B = \{3\}$ so

$$(A \cap B) \cap C = \{3\} \cap \{1,4\} = \phi.$$

Therefore $A \cap B \cap C = \phi$ and there are no elements common to all 3 sets.

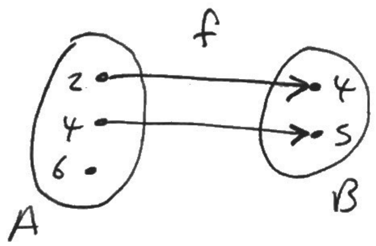
2.3 Functions

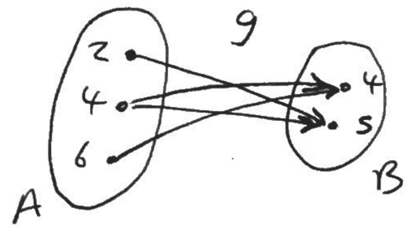
Definition: A <u>function</u> f from set A to set B sends every element of A to a unique element of B.

Then we write $f : \underset{\text{domain}}{A} \longrightarrow \underset{\text{codomain}}{B}$

<u>Example</u> 3 Let $A = \{2, 4, 6\}$, $B = \{4, 5\}$.
Define f by saying $f(2) = 4$, $f(4) = 5$.
Define g by saying $g(2) = 5$, $g(4) = 5$, $g(6) = 4$ and $g(4) = 4$.
Are f and g functions from A to B?

Solution: It is helpful to draw these:

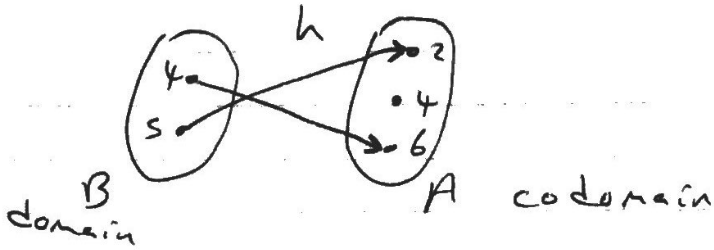




We see clearly that f is <u>not</u> a function from A to B because it doesn't send 6 to an element of B.
Also g is <u>not</u> a function because it doesn't send 4 to a unique element of B.

<u>Example</u> 4 Show that h given by $h(4) = 6$, $h(5) = 2$ is a function from B to A.

Solution:



We see that h does send every element of B to a unique element of A so it is a function from B to A.

For this h its range is $\{2, 6\}$ and not equal to the codomain so h is not onto.

h is one-to-one because it never sends different elements of the domain to the same place.

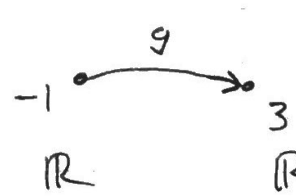
If a function $f: A \to B$ is both onto and one-to-one then it is called a $\boxed{\text{one-to-one correspondance}}$. In that case you can reverse all the arrows and get a new function called the $\boxed{\text{inverse}}$ with notation f^{-1}.

<u>Example</u> 5 The function $g: \mathbb{R} \to \mathbb{R}$ given by $g(x) = 2x + 5$ is a one-to-one correspondance. Find $g(-1)$ and $g^{-1}(1)$.

Solution: $g(-1) = 2(-1) + 5 = -2 + 5 = 3$

So $g(-1) = 3$

input output



If $g^{-1}(1) = x$ then $g(x) = 1$

$$x \xleftarrow{g^{-1}} 1 \qquad x \xrightarrow{g} 1$$

we can solve $g(x) = 1$:

$$2x + 5 = 1 \quad \text{so} \quad 2x = 1 - 5 = -4$$
$$x = -2$$

Therefore $g^{-1}(1) = \underline{-2}$.

We also looked at composition of functions.

$(f \circ g)(x)$ means $f(g(x))$

"f after g" or "composition of f and g".

2.6 Matrices

Example $M = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ is a 2×3 matrix

rows columns.

If N is another 2×3 matrix then we can compute $M + N$ and $M - N$ just by adding, subtracting corresponding entries.

To multiply two matrices is more complicated. You multiply the rows of the first by the columns of the second.

For example with $M = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$, $N = \begin{bmatrix} 0 & 2 \\ -1 & 0 \\ 4 & 5 \end{bmatrix}$

we get

$$MN = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ -1 & 0 \\ 4 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1\cdot 0 + 2(-1) + 3\cdot 4 & 1\cdot 2 + 2\cdot 0 + 3\cdot 5 \\ 4\cdot 0 + 5(-1) + 6\cdot 4 & 4\cdot 2 + 5\cdot 0 + 6\cdot 5 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 17 \\ 19 & 38 \end{bmatrix}$$

$$2 \times \underline{\underline{3}} \cdot \underline{\underline{3}} \times 2 \longrightarrow 2 \times 2 \text{ matrix}$$

sizes

The Boolean product of two zero-one matrices is found the same way – just replace multiplication by $\wedge$ (AND) and addition by $\vee$ (OR).

3.1 Algorithms

These are precise instructions to perform a task. Instead of using a particular programming language, we can write our algorithms in pseudocode.

We studied the simple algorithms:

Max : finds largest integer in a list

Linear search : finds location of a given integer in a list or outputs 0 if not found.

binary search : finds location of a given integer in an ordered list.

bubble sort : puts a list of real numbers into increasing order

insertion sort : uses a different method to put numbers into increasing order.

change : uses a greedy algorithm to to make change, starting with largest value coins.

As in the homework, you could be asked to show all the steps an algorithm uses for a specific input.