

5.3 Percent problems

In section 4.12 we found fractional parts of numbers. For example

$$20\% \text{ of } 40 \text{ means } \frac{20}{100} \cdot 40$$

Convert
Percent to a
number

of means
multiply

$$= \frac{20}{100} \cdot \frac{40}{1} = \frac{800}{100} = 8$$

So we find that 8 is 20% of 40

$$8 = \frac{20}{100} \cdot 40 \text{ true and if}$$

we divide
both sides

by 40 stays
true.

proportion $\rightarrow \frac{8}{40} = \frac{20}{100}$

In general

"A is P percent of B"

means $\frac{A}{B} = \frac{P}{100}$

Same as saying "P percent of B is A". We are thinking of A as a part of the base amount B. The base number usually follows the word "of".

In a percent problem you are given two of the numbers A, B, P and must find the third.

Example (1) 2 is 5% of what number?

Solution: Compare with our template

"A is P percent of B"
↑ ↑ ↑
2 5 missing

$$\frac{A}{B} = \frac{P}{100} \quad \text{becomes} \quad \frac{2}{B} = \frac{5}{100}$$

now we solve this proportion as in section 5.2

(A) Simplify fraction

$$\frac{5}{100} = \frac{5 \div 5}{100 \div 5} = \frac{1}{20}$$

(B) Set cross products equal

$$\frac{2}{B} = \frac{1}{20} \quad 2 \cdot 20 = B \cdot 1$$

$$\text{so } 40 = B$$

(C) If we had an equation with a number multiplying B then divide both sides by that number.

Answer 2 is 5% of 40

Example (2) 9 is what percent of 30?

Solution: "A is P% of B"
 $\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ 9 & \text{missing} & 30 \end{array}$

So $\frac{9}{30} = \frac{P}{100}$

$$\frac{9 \div 3}{30 \div 3} = \frac{3}{10}$$

(A) Simplify $\frac{3}{10} = \frac{P}{100}$

(B) Set cross products equal $3 \cdot 100 = 10P$
 $300 = 10P$

(C) Divide both sides by 10

$$\frac{300}{10} = \frac{10P}{10} \quad \text{means} \quad 30 = P \quad \text{and} \quad P = 30$$

Answer 9 is 30% of 30

Note that P=30 is not a good answer because there is no P in the question.

Example (3) What number is 17% of 20?

Solution: A missing $P=17$ $B=20$

So $\frac{A}{20} = \frac{17}{100}$

Doesn't simplify
 ← Set cross products equal

$$100A = 20 \cdot 17 = 340$$

$$100A = 340$$

To get $1A = A$ divide both sides by 100

$$A = \frac{340}{100}$$

Can simplify this fraction

$$\frac{340 \div 10}{100 \div 10} = \frac{34 \div 2}{10 \div 2} = \frac{17}{5} = 3\frac{2}{5}$$

or (easier) make a decimal

$$\frac{340}{100} \div 100 = 3.4$$

Answer 3.4 is 17% of 20

- More examples p 157.

We can solve more complicated word problems using this method.

Example (4) A baseball team won 93 games, or 62% of the games it played. How many games did the team play?

Solution: Can you see that the question translates to

93 is 62% of what number?

So $A=93$, $P=62$, B missing

$$\frac{93}{B} = \frac{62}{100}$$

(often easier to leave
100 on bottom)

Cross products $93 \cdot 100 = 62B$

$$\begin{array}{r} 9300 \\ \text{"} \\ 9300 \end{array}$$

Divide both sides by 62

$$\frac{9300}{62} = B$$

Long division:

$$\begin{array}{r} 150 \\ 62 \overline{) 9300} \\ \underline{-62} \downarrow \\ 310 \\ \underline{310} \downarrow \\ 00 \\ \underline{-0} \\ 0 \end{array}$$

So $B=150$

but answer word

problem question:

Answer the team played 150 games

Check the answer makes sense

93 is 62% of 150 ✓

answer had to be bigger than 93 for sure

75 is 50% of 150 similar.

Example (5) Marina's annual salary last year was \$56000. This year she received a raise of \$4480. By what percent did her salary increase?

Solution: In this question we are comparing the raise (increase) of 4480 with the base salary 56000.

So $A = 4480$, $B = 56000$, P missing

$$\frac{4480}{56000} = \frac{P}{100}$$

$$448 \cdot 100 = 5600 P$$

$$P = \frac{44800}{5600} = \frac{448}{56}$$

Can simplify $\frac{448 \div 2}{56 \div 2} = \frac{224 \div 2}{28 \div 2} = \frac{112 \div 2}{14 \div 2} = \frac{56}{7} = 8$

So $P = 8$

Answer Her salary increased by 8%

- Examples p157, 158.