

Chapter 4 Decimals and Percents

We see decimals everyday, for example on receipts or credit card statements:

balance	\$ 943.65
APR	14.99%
Sales tax	8.875%

We'll look at percents later, but what do the basic decimals like 943.65 mean?

4.1 Decimal place values

Decimals give a convenient way to write fractions and mixed numbers using tens.

Powers of 10: $10^2 = 10 \cdot 10 = 100$
 $10^3 = 10 \cdot 10 \cdot 10 = 1000$
:
 $10^6 = \underbrace{1000000}_{6 \text{ zeros}}$

$943.\underline{65}$ means $943 \frac{65}{100}$ mixed number
2 places $\rightarrow 10^2$ denominator

$16.\underline{104}$ means $16 \frac{104}{1000}$
3 places $\rightarrow 10^3$ denominator

So one way to understand decimals is to see that the decimal point separates the whole number part from the fraction part:

$$\begin{array}{c} \underline{3}.\underline{41} \\ \text{whole} \quad \text{fraction} \\ \text{number} \end{array} = 3 \frac{41}{100}$$

$$100.023 = 100 \frac{023}{1000} = 100 \frac{23}{1000}$$

Note that $3.41 = 3 \frac{41}{100}$

but $3.41 = 123$

Example ① Express these decimals as mixed numbers:

(a) 20.9 (b) 8.875 (c) 0.04

Solution: (a) $20.9 = 20 \frac{9}{10}$ (1 place, $10^1 = 10$)

(b) $8.875 = 8 \frac{875}{1000}$ (3 places, $10^3 = 1000$)

(c) $0.04 = \frac{04}{100} = \frac{4}{100}$ (2 places, $10^2 = 100$)

Parts (b) and (c) can also be simplified

$$\frac{4}{100} = \frac{4 \div 4}{100 \div 4} = \frac{1}{25}$$

So 0.04 also equals $\frac{1}{25}$.

Example (2) Write these mixed numbers as decimals:

(a) $4\frac{7}{10}$ (b) $2\frac{99}{10000}$ (c) $\frac{23}{100}$

Answers: (a) 4.7 (b) $2.\underbrace{0099}_{\substack{\text{needs} \\ \text{4 places}}}$ (c) 0.23

A second way to understand decimals looks at each digit. For example

98.76 contains digits 9, 8, 7, 6

and the 9 represents 9 tens, 9×10

8 represents 8 ones, 8×1

7 represents 7 tenths $7 \times \frac{1}{10}$

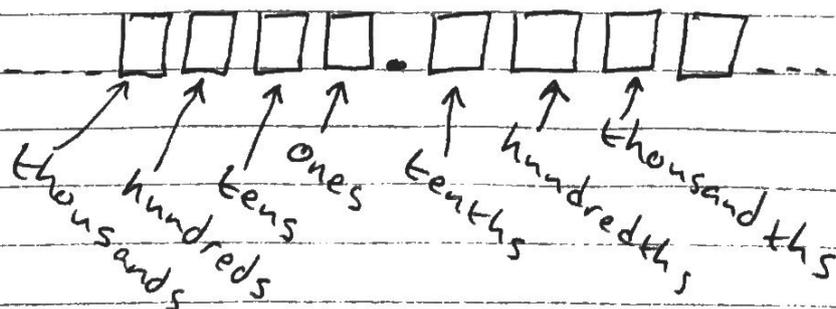
6 represents 6 hundredths $6 \times \frac{1}{100}$

Altogether $98.76 = 9 \times 10 + 8 \times 1 + 7 \times \frac{1}{10} + 6 \times \frac{1}{100}$

$$= 90 + 8 + \frac{7}{10} + \frac{6}{100}$$

$$= 98 + \frac{70}{100} + \frac{6}{100} = 98\frac{76}{100}$$

Decimal place value system



Remember that the decimal point is always between the ones and the tenths.

"Decimal" means tenth in Latin - everything is based on tens

$$\text{one thousand} = 1000 = 10^3$$

$$\text{one hundred} = 100 = 10^2$$

$$\text{one ten} = 10 = 10^1$$

$$\text{one} = 1 = 10^0$$

$$\text{one tenth} = \frac{1}{10} = \frac{1}{10^1} = 10^{-1}$$

$$\text{one hundredth} = \frac{1}{100} = \frac{1}{10^2} = 10^{-2}$$

$$\text{one thousandth} = \frac{1}{1000} = \frac{1}{10^3} = 10^{-3}$$

Example (3) For this decimal

904.328

name the digit in (a) the tens place and (b) the hundredths place.

↑
negative powers
mean reciprocals
(one over).

Answer: (a) Digit 0 in tens place,

(b) Digit 2 in hundredths place.

Note that every whole number can be written as a decimal. For example, do you see where the decimal point goes for this number?

65

- More examples p 120.

4.2 Significant and insignificant zeros

Insignificant zeros are ones you can remove from a decimal without changing its value.

For example 00340.9060

has three insignificant zeros and simplifies to

340.906

The two remaining zeros are significant:

$340.906 \neq 34.96$

Rule

Zeros to the left of the whole number part or the right of the fraction part are insignificant. Other zeros are significant.

Example (4) Simplify these decimals by removing all insignificant zeros

(a) $042.900 = 42.9$

(b) $30600.0 = 30600$

(c) $00.009 = 0.009$ or $.009$

(d) $570.0306 = 570.0306$

(e) $00900 = 900$

So, to round 924.83 to the nearest 10 we first find the tens place (called the round off place)

$$9\boxed{2}4.83$$

and then look at the place to the right which has a 4 in it. Since $4 < 5$ (middle) we keep the 2 in the round off place. Then make the 4 and every digit to the right zero and simplify

$$9\boxed{2}4.83 \rightarrow 9\boxed{2}0.00 = 920$$

Example (6) Round 927.83 to the nearest 10.

Similar solution: $9\boxed{2}7.83$

round off place

$7 \geq 5$ (middle)

so increase digit in round-off place

$$9\boxed{2}7.83 \rightarrow 9\boxed{3}0.00 = 930$$

So 927.83 rounded to nearest 10 is $\boxed{930}$.

Rule: If digit to right of round off place is 0 to 4 then keep round off digit.

If digit to right of round off place is 5 to 9 then increase round off digit by one.

Then make all the digits to the right of the round off place zero and simplify.

Example (7) Round 413.098 to
(a) nearest hundredth, (b) nearest hundred.

Solution: Part (a) 413.098
round off place \nearrow \nwarrow in range 5 to 9

We must increase the 9 by one

$$413.0\boxed{9}8 \rightarrow 413.100 = \boxed{413.1}$$

↑ ↑
increase make zero

Part (b) $\boxed{4}$ 13.098
round off place \nearrow \nwarrow in range 0 to 4

$$\boxed{4} \underbrace{13.098}_{\text{make zero}} \rightarrow 400.000 = \boxed{400}$$

↑
keep

• Examples p 124, 125.