

Mth 31, Homework 4 on sections 3.1, 3.2, 3.3

Due by Mon, Oct 7.

Try these 15 questions. Write all your working out and answers by hand on your own notepaper and hand them to me next week. Please use lots of space and as many pages as you want, so I can include corrections or comments. You do not need to write the questions, but it is very important that you show clearly any work you had to do to get your answers. Each question is worth 3 points.

For the following questions use the differentiation formulas

$$\frac{d}{dx}c = 0, \quad \frac{d}{dx}x^n = nx^{n-1}, \quad \frac{d}{dx}e^x = e^x, \quad \frac{d}{dx}\sin x = \cos x, \quad \frac{d}{dx}\cos x = -\sin x$$

and the sum, difference, product and quotient rules.

Section 3.1 Derivatives of polynomials and exponentials

(1) Differentiate: $f(x) = 3x^4 - \frac{2}{3}x^3 + \sqrt{5}x + 16$

(2) Differentiate: $g(x) = x^{2/3} - \frac{1}{x^2} + 4\sqrt{x}$

(3) Differentiate: (a) $f(x) = 13e^x - 12$ (b) $h(t) = t^4 + \sqrt{3} + t + e^4$

(4) Suppose $y = 4t^2x + tx^5$ and find: (a) $\frac{dy}{dx}$ (b) $\frac{dy}{dt}$

(5) Let $y = x + \frac{2}{x}$

(a) Find the equation of the tangent line to this curve at the point (2, 3).

(b) Find the equation of the normal line to this curve at (2, 3). (The normal line is perpendicular to the tangent line.)

(6) Let $s(t) = t^3 - 9t^2 + t$ give the position of an object in meters after t seconds.

(a) Find its velocity $v(t)$.

(b) Find its acceleration $a(t)$.

(c) How fast is the object moving after 10 seconds?

(d) When does the object have zero acceleration?

Section 3.2 The product and quotient rules

(7) Let $g(x) = (x^3 + 1)(x^2 - 1)$.

- (a) Find $g'(x)$ using the product rule for differentiation and simplifying.
- (b) Find $g'(x)$ by first multiplying out the product and then differentiating.
- (c) Check that you get the same answer in (a) and (b).

(8) Let $f(x) = x^3 e^x$ and find: (a) $f'(x)$ (b) $f''(x)$

(9) Suppose

$$g(x) = xf(x), \quad f(4) = -2, \quad f'(4) = 3$$

and use this information to find $g'(4)$.

(10) Use the quotient rule to differentiate $\frac{x+2}{x+3}$

Section 3.3 Derivatives of trigonometric functions

(11) Compute:

(a) $\frac{d}{dx} (4 \sin x - 3 \cos x + 2)$

(b) $\frac{d}{dt} (\sin t \cos t)$

(c) $\frac{d}{d\theta} (\cos^2 \theta)$

(d) $\frac{d}{dx} (5 \tan x)$

(Remember that $\cos^2 \theta$ means $(\cos \theta)^2$.)

(12) Let

$$f(\theta) = \frac{\sin \theta}{1 + \cos \theta}$$

and find $f'(\theta)$

(13) What is the equation of the tangent line to $y = x + 1 + \sin x$ at the point $(0, 1)$.

(14) Let $h(x) = x \sin x$ and find $h^{(4)}(x)$. This is the fourth derivative.

(15) Estimate

$$\lim_{x \rightarrow 0} \frac{x}{\sin x}$$

numerically by using some x values close to 0. (Make sure your calculator is in radians mode.)

If you get stuck on a question or aren't sure if you understand it:

- Go over the relevant class notes and section in the textbook.
- Check if you get the right answer for a similar odd-numbered question in the textbook (answers at the back of the book).
- Ask me about it after class.
- Come to my office hours: Mon 12:00 - 1:00, Wed 12:00 - 1:00 in CP 317.
- Go to the Math Tutorial Lab in-person in CP 303 or online.