

Mth 31, Homework 12 on sections 5.2, 5.3

Due by Wed, Dec 4.

Each question is worth 3 points. Please use lots of space and as many pages as you want, so I can include corrections or comments.

We saw these basic properties of definite integrals:

$$\begin{aligned}\int_a^a f(x) dx &= 0 & \int_a^b f(x) dx + \int_b^c f(x) dx &= \int_a^c f(x) dx \\ \int_b^a f(x) dx &= -\int_a^b f(x) dx & \int_a^b 1 dx &= b - a \\ \int_a^b cf(x) dx &= c \int_a^b f(x) dx & \int_a^b (f(x) \pm g(x)) dx &= \int_a^b f(x) dx \pm \int_a^b g(x) dx\end{aligned}$$

Section 5.2 The definite integral

(1) Use these properties to find: (a) $\int_2^2 \tan(x) dx$ (b) $\int_{-2}^6 4 dx$ (c) $\int_6^{-2} 4 dx$

(2) Find

$$\int_0^{\pi/2} (3 + 8 \sin^4 x) dx + \int_{\pi/2}^{\pi} (7 + 8 \sin^4 x) dx$$

if we are given that $\int_0^{\pi} \sin^4 x dx = \frac{3\pi}{8}$.

The Fundamental Theorem of Calculus

Suppose $f(x)$ is continuous for x in $[a, b]$. Then Part 1 of this theorem says

$$g'(x) = f(x) \quad \text{if } g \text{ is given by } g(x) = \int_a^x f(t) dt.$$

Part 2 says

$$\int_a^b f(t) dt = F(b) - F(a) \quad \text{for } F \text{ any antiderivative of } f.$$

Section 5.3 The Fundamental Theorem of Calculus

(3) Use part 2 of the theorem to calculate: $\int_0^{16} \sqrt{t} dt$

(4) Use part 2 to find: $\int_1^4 (4 - 2x) dx$

(Hint: use properties of integrals to break this up into simpler pieces and then find their antiderivatives.)

(5) Compute $\int_0^3 (4x^2 - 5) dx$ by using antiderivatives.

(6) Find: $\int_0^{3\pi} (1 + \cos \theta + 4e^\theta) d\theta$

(7) Use part 2 to compute:

$$\int_1^4 \frac{2 + x^2}{\sqrt{x}} dx$$

(8) Is the evaluation $\int_{-2}^2 \frac{3}{x^4} dx = -\frac{1}{4}$ correct? What could the problem be?

(9) Use part 1 of the theorem to find the derivative of

$$g(x) = \int_1^x \sqrt{t^2 + 1} dt$$

(10) Use part 1 and the chain rule to work out: $\frac{d}{dx} \int_0^{x^3} \sqrt{2t + 1} dt$

(11) Find: $\frac{d}{dx} \int_x^{10} (\sin(t^3) + \ln(t^4)) dt$

If you get stuck on a question or aren't sure if you understand it:

- Go over the relevant class notes and section in the textbook.
- Check if you get the right answer for a similar odd-numbered question in the textbook (answers at the back of the book).
- Ask me about it after class.
- Come to my office hours: Mon 12:00 - 1:00, Wed 12:00 - 1:00 in CP 317.
- Go to the Math Tutorial Lab in-person in CP 303 or online.