Math 31, Practice Final Exam

This practice final is to give you an idea of what to expect and help us review. Any questions similar to quiz or homework questions can appear.

On the final you will be asked to do any 15 of the 18 questions in 2 hours and 50 minutes. They are worth 6 points each. To get all 6 points it is very important that you show clearly all your working out and reasoning. You may use your own calculator. Calculators may not be shared. Phones and other technology must be put away - any use of a phone or smart watch will result in an F grade.

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(1) Let $f(x)$ be function defined by

$$f(x) = \begin{cases} 1-x & \text{if } x < 1 \\ x & \text{if } x \geq 1. \end{cases}$$

- **(a)** Sketch the graph of $f(x)$.
- **(b)** Does $\lim_{x\to 1} f(x)$ exist?
- **(c)** Explain why $f(x)$ is continuous or not at $x = 1$.

(2) Compute these limits exactly:

(a) $\displaystyle\lim_{x\to 0} \frac{\sin 5x}{6x}$ **(b)** $\displaystyle\lim_{x\to -\infty} \frac{4x^3 - x^2 - 4}{x^3 + x}$ **(c)** $\displaystyle\lim_{x\to\infty} \frac{-x^3 + 15x^2 + 7}{10x^2 - 9}$

(3) Let $f(x) = x^4 - 3x + 1$.

- **(a)** Explain why the *Intermediate Value Theorem* shows that there is at least one solution to the equation $f(x) = 0$ in the interval $(-1, 1)$.
- **(b)** What exactly does the *Mean Value Theorem* say about $f(x)$ on the interval $[-1, 1]$?

(4) Let $g(x) = \sqrt{x+1}$.

- **(a)** Use the limit definition of derivative to find $g'(x)$
- **(b)** For which real numbers is your formula for part (a) true? Explain.

(5) Compute these derivatives:

(a) $\displaystyle\frac{d}{dx}(x^3+1)(x^2-1),$ **(b)** $\displaystyle\frac{d}{d\theta}\frac{\sin\theta}{1+\cos\theta},$ **(c)** $\displaystyle\frac{d}{dx}\cos\left(\frac{1}{x^5}\right)$

(6) Find the equation of the tangent line at $(1, 2)$ to the curve

$$x^3 + xy + y^3 = 11$$

(7) A ball is thrown vertically up and its height after t seconds is

$$s(t) = 48t - 4t^2 \quad \text{meters.}$$

Answer these questions using the correct units.

(a) Find the velocity of the ball: $v(t)$

(b) Find the acceleration of the ball: $a(t)$

(c) When does the ball have zero velocity?

(d) Find the maximum height of the ball.

(e) Find the velocity of the ball as it hits the ground.

(8) A plane is flying horizontally at a height of 1 mile and speed of 500 mi/h. It passes directly over a radar station. Find the rate at which the distance of the plane from the radar is increasing when this distance is 2 miles.

(9) Use a linear approximation to estimate: $\sqrt[3]{1001}$

(10) Find the absolute maximum and minimum of $g(x) = 2x^3 - 3x^2 - 12x + 1$ on $[-2, 3]$.

(11) Let $f(x) = \dfrac{x}{x^2 - 4}$.

(a) Find the domain of f.

(b) Find all x and y intercepts.

(c) Is f odd or even?

(d) Find all vertical and horizontal asymptotes.

(e) Give the intervals where f is increasing and decreasing.

(f) Find all the local maximums and minimums: identify which is which and give their coordinates.

(12) Let $f(x) = \dfrac{x}{x^2 - 4}$ as in the previous question.

(a) Find where f is concave up or down and locate any inflection points.

(b) Graph $f(x)$ using all the information you have found in part (a) of this question and parts (a) - (f) of the previous question, plotting any extra points you need.

(13) A plastic bucket in the shape of a circular cylinder with a base but no top is being designed to contain a volume of $8000\pi \ cm^3$. What should the dimensions be to minimize the amount of plastic.

(14) Use Newton's method to estimate $\sqrt[4]{18}$ correct to at least 3 decimal places.

(15) Use a Riemann sum with $n = 4$ rectangles, taking the sample points to be midpoints, to estimate:

$$\int_0^8 \sqrt{x^3 + 1}\, dx$$

(16) Compute $\displaystyle\int_0^4 (3x^2 - 5)\,dx$ by using its definition as a limit of Riemann sums (using right end points).

(17) Use the two parts of the *Fundamental Theorem of Calculus* to:

(a) Find $\displaystyle\frac{d}{dx}\int_3^{x^2} \sin^3(t)\,dt$

(b) Compute $\displaystyle\int_0^4 (3x^2 - 5)\,dx$

(18) Evaluate these definite and indefinite integrals:

(a) $\displaystyle\int_1^4 \frac{2 + x^2}{\sqrt{x}}\,dx,$ **(b)** $\displaystyle\int \sec^2(7x + \pi)\,dx,$ **(c)** $\displaystyle\int x^3\sqrt{x^2 + 1}\,dx$