Math 31, Homework 9 on sections 4.2, 4.3

Do these 12 questions and *check that your answers match the solutions on page 2*. They will not be collected, but similar questions could appear on the next quiz and on the final. Let me know if you are having any difficulty doing them.

(1) Use a Riemann sum with $n = 4$ rectangles, taking the sample points to be midpoints, to estimate:

$$\int_0^4 \sqrt{x^3+1}\,dx$$

(2) Write the definition of $\displaystyle\int_2^5 (4-2x)\,dx$ as a limit of Riemann sums (using right end points).

(3) Use the formulas

$$\sum_{i=1}^{n} 1 = n, \quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

to find the limit in Question 2.

(4) Compute $\displaystyle\int_0^4 (3x^2-5)\,dx$ by using its definition as a limit of Riemann sums (using right end points) along with the formula

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

(5) Graph the line $y = 2x - 2$. Use this graph and the areas of the triangles it makes to find:

$$\int_0^3 (2x-2)\,dx$$

(6) Use part 1 of the Fundamental Theorem of Calculus to find the derivative of

$$g(x) = \int_1^x \sqrt{t^2+1}\,dt$$

(7) Find the derivative of

$$h(x) = \int_3^{x^2} \sin^3(t)\,dt$$

(8) Use part 2 of the Fundamental Theorem of Calculus to calculate: $\displaystyle\int_2^5 (4-2x)\,dx$

(9) Compute $\displaystyle\int_0^4 (3x^2-5)\,dx$ by using an antiderivative.

(10) Find: $\int_0^{16} \sqrt{u}\, du$

(11) Find: $\int_0^{3\pi} (1 + \cos(\theta))\, d\theta$

(12) Is the evaluation $\int_{-2}^{2} \frac{3}{x^4}\, dx = -\frac{1}{4}$ correct? What could the problem be?

Answers to questions (1)-(12):

(1) 13.8535

(2) Let $f(x) = 4 - 2x$. Then $\int_2^5 (4 - 2x)\, dx = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(2 + \frac{3i}{n}\right) \frac{3}{n}$

(3) This limit is -9

(4) 44

(5) The integral equals $-1 + 4 = 3$

(6) $g'(x) = \sqrt{x^2 + 1}$

(7) $h'(x) = 2x \sin^3(x^2)$

(8) -9

(9) 44

(10) $128/3$

(11) 3π

(12) We cannot use part 2 of the Fundamental Theorem because $3/x^4$ is not defined at $x = 0$. (In fact the area under this graph between $x = -2$ and $x = 2$ is infinite.)