Math 31, Homework 8 on sections 3.8, 3.9, 4.1, 4.2

Write all your working out and answers on your own notepaper - no need to write the questions. Please use lots of space.

It is very important that you show clearly any work you had to do to get your answers. Just writing the answer down with no work shown is usually not enough. Do all 15 questions - they are worth 2 points each. Hand in your solutions next week only.

For these first 10 questions, *check that your answers match the solutions on page 2*. If you don't get the same answer then go back and see where you went wrong.

(1) Use Newton's method to approximate a solution to $x^2 - 5 = 0$ as follows:

- **(a)** Starting with $x_1 = 2$, find the second approximation x_2.
- **(b)** Then find the third approximation x_3.
- **(c)** Compare your answers with the decimal of the exact solution.

(2) Use Newton's method to estimate $\sqrt[3]{25}$ correct to at least 3 decimal places.

(3) Find the most general antiderivative of

$$6x^5 - 8x^4 + 3x^3$$

and check that your answer works by differentiating.

(4) Find $f(x)$ if

$$f'(x) = x^7 + 4 + \sin x \qquad \text{and} \qquad f(0) = 3$$

(5) Find the most general antiderivative of:

$$x^{2/5} - \frac{2}{x^4} + x\sqrt{x} - 4\sec^2(x)$$

(6) Let $f(x) = 1 + \sin(x)$. Use three rectangles and right endpoints to estimate the area under this graph between $x = 0$ and $x = \pi/2$. Draw a diagram showing the graph and rectangles.

(7) Let $f(x) = x^{-2}$. Use two rectangles and midpoints to estimate the area under this graph between $x = 1$ and $x = 5$. Draw a diagram showing the graph and rectangles.

(8) The velocity $v(t)$ of a car is measured every half hour, giving the following data (t is measured in hours and $v(t)$ measured in miles per hour):

$$v(0.5) = 55, \quad v(1) = 40, \quad v(1.5) = 50, \quad v(2) = 60, \quad v(2.5) = 65$$

Approximately how far did the car travel between $t = 0$ and $t = 2.5$?

(9) Write the area under the graph of $f(x) = x^3$ between $x = 0$ and $x = 2$ as a limit (using right endpoints). Do not evaluate the limit.

(10) Write the following limit, on the interval $[0, 2\pi]$, as a definite integral:

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\cos(x_i)}{x_i^2 + 1} \Delta x$$

Five more questions. Show clearly all your working out and reasoning. Only do these questions when you are sure you understand the first ten.

(11) Use Newton's method to estimate $\sqrt[4]{4}$ correct to at least 3 decimal places.

(12) Find $f(x)$ if

$$f''(x) = 20x^3 - 8 \qquad \text{and} \qquad f(0) = 5,\ f(1) = 0$$

(13) Let $f(x) = 2 + \sqrt{x}$. Use three rectangles and midpoints to estimate the area under this graph between $x = 0$ and $x = 3$. Draw a diagram showing the graph and rectangles.

(14) Write the area in Question 13 as a limit (using right endpoints). Do not evaluate the limit.

(15) Write the area in Question 13 as a definite integral.

Answers to questions (1)-(10):

(1) (a) $x_2 = 2.25$, (b) $x_3 = 2.23611$, (c) x_3 is close to the exact solution $\sqrt{5} \approx 2.23608$

(2) With $x_1 = 3$ we find x_3 and x_4 agree to three places, giving 2.924. So with this value Newton's method has found $\sqrt[3]{25}$ correct to three places.

(3) $x^6 - \frac{8}{5}x^5 + \frac{3}{4}x^4 + C$

(4) $f(x) = \frac{1}{8}x^8 + 4x - \cos(x) + 4$

(5) The most general antiderivative is

$$\frac{5}{7}x^{7/5} + \frac{2}{3}x^{-3} + \frac{2}{5}x^{5/2} - 4\tan(x) + C$$

(6) The estimate is $\frac{\pi}{6}\left(3 + \sin(\pi/6) + \sin(2\pi/6) + \sin(3\pi/6)\right) \approx 2.80964$

(7) The area estimate is $5/8 = 0.625$

(8) The car traveled approximately 135 miles.

(9) $\lim_{n\to\infty} \frac{16}{n^4} \sum_{i=1}^{n} i^3$

(10) $\int_0^{2\pi} \frac{\cos(x)}{x^2 + 1}\, dx$