Math 31, <u>Extra Credit</u> Homework 7 on sections 3.3, 3.4, 3.5, 3.7

Write all your working out and answers on your own notepaper - no need to write the questions. Please use lots of space.

It is very important that you show clearly any work you had to do to get your answers. Just writing the answer down with no work shown is usually not enough. Do all 15 questions - they are worth 2 points each. Hand in your solutions next week only.

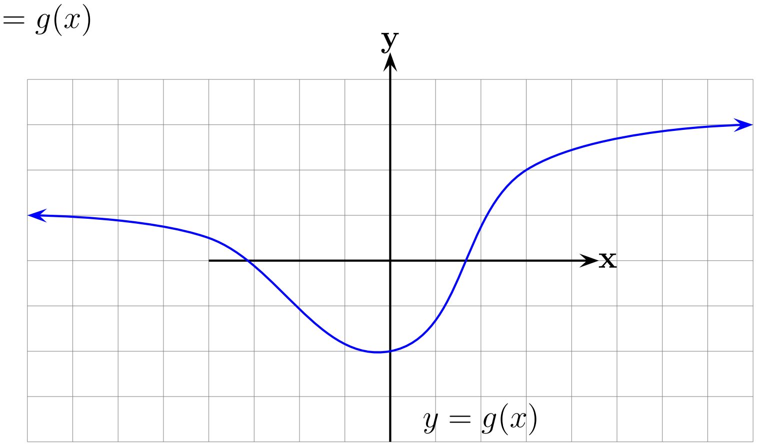
For these first 10 questions, *check that your answers match the solutions on page 4*. If you don't get the same answer then go back and see where you went wrong.

(1) Let $f(x) = -x^2 + 2x + 15$.

(a) Use the first derivative to see the xs where f is increasing and decreasing. Give your answer in interval notation.

(b) Find all the local maximums and minimums: identify which is which and give their coordinates.

(2) For this graph $y = g(x)$



(a) Give the intervals where g is increasing and decreasing.

(b) Identify and locate all local maximums and minimums.

(c) Give the intervals where g is concave up and down.

(d) Locate all inflection points.

(3) Let $f(x) = x^3 - 12x^2 - x$.

(a) Use the second derivative to see the xs where f is concave up and down. Give your answer in interval notation.

(b) Locate all inflection points.

(4) Let $f(x)$ be the rational function $\dfrac{-x^2-5x+2}{4x^2-2}$. Find

$$(a) \quad \lim_{x\to\infty} f(x) \qquad\qquad (b) \quad \lim_{x\to-\infty} f(x)$$

(c) How do the results from parts (a) and (b) show up on the graph of $f(x)$?

(5) Compute

$$\lim_{x\to\infty} \frac{\sqrt{4x^2+1}}{4+x^2}$$

(Hint: you might need to use that $x^2 = \sqrt{x^4}$ and $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$.)

(6) Let $f(x) = \dfrac{x}{x^2-4}$.

(a) Find the domain of f.

(b) Find all x and y intercepts.

(c) Is f odd or even? (Odd means $f(-x) = -f(x)$ and even means $f(-x) = f(x)$.)

(d) Find all vertical and horizontal asymptotes.

(e) Give the intervals where f is increasing and decreasing.

(f) Find all the local maximums and minimums: identify which is which and give their coordinates.

(g) Find where f is concave up or down and locate any inflection points.

(7) Graph $f(x)$ from question 6 using parts (a)-(g) and plotting any extra points you need.

(8) Find two numbers with sum 18 and product as large as possible.

(9) Find the point on the line $y = 1 + 2x$ that is closest to the point $(5, 1)$. (Hint: minimize the square of the distance between the points.)

(10) A farmer wants to enclose an area of 6 square miles with a rectangular fence. The fence should also divide the area in two with a line of fence running parallel to two sides. What is the shortest length of fence the farmer can use? Draw a diagram of this shortest fence.

Five more questions. Show clearly all your working out and reasoning. Only do these questions when you are sure you understand the first ten.

(11) For the graph of $\sin x$ on the interval $[0, 2\pi]$, find the intervals where it is increasing/decreasing and where it is concave up/down.

(12) Use parts (a)-(g) of Question 6 to describe the graph of

$$f(x) = \frac{3}{x^4 + 1}$$

Use this information to draw a detailed, labelled graph of f.

(13) Find:

$$(a) \quad \lim_{x \to -\infty} \frac{4x^3 - x^2 - 4}{x^3 + x} \qquad (b) \quad \lim_{x \to \infty} \frac{-x^3 + 15x^2 + 7}{10x^2 - 9} \qquad (c) \quad \lim_{x \to \infty} \cos\left(\frac{1}{x}\right)$$

(14) Find the point on the line $y = -3x - 3$ that is closest to the point $(3, -2)$.

(15) A plastic bucket in the shape of a circular cylinder with a base but no top is being designed to contain a volume of $8000\pi\ cm^3$. What should the dimensions be to minimize the amount of plastic.

Answers to questions (1)-(10):

(1) (a) The graph is increasing on the interval $(-\infty, 1)$ and decreasing on $(1, \infty)$.
(b) The only local extreme is a local maximum at the point $(1, 16)$.

(2) (a) The graph is increasing on the interval $(0, \infty)$ and decreasing on $(-\infty, 0)$.
(b) The only local extreme is a local minimum at the point $(0, -2)$.
(c) The graph is concave up on the interval $(-2, 1.6)$ approximately, and concave down on $(-\infty, -2) \cup (1.6, \infty)$.
(d) There are inflection points at $(-2, -1)$ and $(1.6, 0)$.

(3) (a) The graph is concave up on the interval $(4, \infty)$ and concave down on $(-\infty, 4)$.
(b) There is an inflection point at $(4, -132)$.

(4) (a) $-1/4$ (b) $-1/4$ (c) The results from parts (a) and (b) mean that the graph of $f(x)$ approaches the horizontal asymptote $y = -1/4$ on the left and right.

(5) 0

(6) (a) The domain is all real numbers except -2 and 2. In interval notation: $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$
(b) $(0, 0)$ is the only point the graph crosses the axes.
(c) The function is odd so the graph has symmetry about the origin.
(d) The asymptotes are the lines: $x = -2$, $x = 2$, $y = 0$
(e) $f'(x) = \dfrac{-x^2 - 4}{(x^2 - 4)^2}$ so f is always decreasing on its domain.
(f) No local extreme points.
(g) $f''(x) = \dfrac{2x(x^2 + 12)}{(x^2 - 4)^3}$ so f is concave up on $(-2, 0) \cup (2, \infty)$ and concave down $(-\infty, -2) \cup (0, 2)$. The only inflection point in the domain is $(0, 0)$.

(7) To draw the graph, first draw the three asymptotes as dashed lines. Plot the point $(0, 0)$ and also plot the points on each side of the asymptotes when $x = -3, -1, 1, 3$. Draw the graph through these points and getting close to the asymptotes. Make sure it is increasing, decreasing in the right places, has the correct concavity and has symmetry about the origin.

(8) The numbers are 9 and 9.

(9) The closest point is $(1, 3)$.

(10) The farmer needs 12 miles of fence. Draw the arrangement.