Math 31, Homework 6 on sections 3.1, 3.2

Do these 12 questions and *check that your answers match the solutions on page 2*. They will not be collected, but similar questions could appear on the next quiz and on the final. Let me know if you are having any difficulty doing them.

(1) Let $f(x) = x^3 - 2x - 4$. Find the values of this function

- **(a)** when $x = 2$
- **(b)** when $x = -3$
- **(c)** when $x = 1/2$

(2) Which of the numbers -2, 1.4, -1, 6, 3.001, -0.99 are in the closed interval $[-1, 3]$?

(3) Sketch the graph of $\cos x$ for x in the closed interval $[0, 5\pi/2]$. Use your graph to find the absolute maximum and minimum values of the function on this interval and identify all local maximums and minimums.

(4) Sketch the graph of $f(x) = x^2 - 1$ for x in the closed interval $[-1, 2]$. Use your graph to find the absolute maximum and minimum values of the function on this interval and identify all local maximums and minimums.

(5) Use the closed interval method to find the absolute maximum and minimum values of $f(x) = 12 + 4x - x^2$ on the closed interval $[0, 5]$.

(6) Find the absolute maximum and minimum of $g(x) = 2x^3 - 3x^2 - 12x + 1$ on $[-2, 3]$.

(7) Find the absolute maximum and minimum of

$$h(x) = \frac{x}{x^2 - x + 1} \quad \text{on} \quad [0, 3]$$

(8) Your flight from New York to Los Angeles takes 6 hours to go 2400 miles. What is your average velocity? What does the *Mean Value Theorem* say about your flight?

(9) Let $f(x) = -2x^2 + x + 1$.

- **(a)** Verify that $f(x)$ satisfies the hypotheses of the Mean Value Theorem on $[-1, 3]$.
- **(b)** Find all the numbers c that satisfy the conclusion of the Mean Value Theorem.

(10) Let $g(x) = x^3 - 3x + 2$.

- **(a)** Verify that $g(x)$ satisfies the hypotheses of the Mean Value Theorem on $[-2, 2]$.
- **(b)** Find all the numbers c that satisfy the conclusion of the Mean Value Theorem.

(11) **(a)** Verify that $\sin x$ satisfies the hypotheses of the Mean Value Theorem on $[0, 2\pi]$.

- **(b)** Find all the numbers c that satisfy the conclusion of the Mean Value Theorem.

(12) Show that the equation

$$x^3 - 15x + 1 = 0$$

has at most one solution in the interval $[-2, 2]$. (Hint: suppose it has two solutions a and b in the interval with $a < b$. Then apply the Mean Value Theorem to $f(x) = x^3 - 15x + 1$ on $[a, b]$. Show that this leads to an impossible situation. Therefore the equation cannot have two different solutions and can have at most one.)

Answers to questions (1)-(12):

(1) (a) $f(2) = 0$, (b) $f(-3) = -25$, (c) $f(1/2) = -39/8 = -4.875$

(2) Only the numbers 1.4, -1 and -0.99 are in this interval.

(3) Sketch the graph. The absolute maximum value is 1 and the absolute minimum value is -1. The graph has a local minimum value of -1 near $x = \pi$ and it has a local maximum value of 1 near $x = 2\pi$.

(4) Sketch the graph for x between -1 and 2. It is the usual parabola moved down one unit. The absolute maximum value is 3 and the absolute minimum value is -1. The graph has a local minimum value of -1 near $x = 0$ and no local maximum.

(5) The absolute maximum is 16 and the absolute minimum is 7.

(6) The absolute maximum is 8 and the absolute minimum is -19.

(7) This function is defined and continuous on $[0, 3]$ because the denominator is never 0. The absolute maximum is 1 and the absolute minimum is 0.

(8) Average velocity is $400\ mi/h$. The Mean Value Theorem says that at some point in your flight the plane has an instantaneous velocity of exactly $400\ mi/h$.

(9) **(a)** Since $f(x)$ is a polynomial, it is continuous and differentiable for all real numbers x.

(b) $c = 1$

(10) **(a)** Since $g(x)$ is a polynomial, it is continuous and differentiable for all real numbers x.

(b) $c = \pm\frac{2\sqrt{3}}{3}$

(11) **(a)** Trigonometric functions are continuous and differentiable for all real numbers in their domain. Therefore $\sin x$ is continuous and differentiable on $[0, 2\pi]$.

(b) $c = \pi/2$ or $c = 3\pi/2$

(12) Following the hint, the Mean Value Theorem says there is a number c in the interval (a, b) where $f'(c) = 0$. Therefore $3c^2 - 15 = 0$ and $c = \pm\sqrt{5}$. But these c numbers are outside $[-2, 2]$ and so outside (a, b). This contradiction shows that there cannot be two solutions.