Math 31, Homework 5 on sections 2.8, 2.9 plus more differentiation formulas

---

Write all your working out and answers on your own notepaper - no need to write the questions. Please use lots of space.

It is very important that you show clearly any work you had to do to get your answers. Just writing the answer down with no work shown is usually not enough. Do all 15 questions - they are worth 2 points each. Hand in your solutions next week only.

For these first 10 questions, *check that your answers match the solutions on page 2*. If you don't get the same answer then go back and see where you went wrong.

(1) Find: $\dfrac{d}{dx}\left(x^5 \tan x\right)$

(2) Find: $\dfrac{d}{dx}\left(\dfrac{\tan x}{x^4}\right)$

(3) Compute: $\dfrac{d}{dx}\cos\left(x^{12}\right)$

(4) The length of a rectangle is increasing at 4 cm/s and its width is increasing at 3 cm/s. Find the rate of change of the area of the rectangle when the length is 20 cm and the width is 10 cm.

(5) Suppose $x^3 + y^3 = 28$. If $\dfrac{dx}{dt} = 4$ then find $\dfrac{dy}{dt}$ when $x = 1$ and $y = 3$.

(6) A plane is flying horizontally at a height of 1 mile and speed of 500 mi/h. It passes directly over a radar station. Find the rate at which the distance of the plane from the radar is increasing when this distance is 2 miles. (Bonus question - can you explain why this rate is less than 500 mi/h?)

(7) A 10 foot ladder is resting against a wall but the bottom is sliding out at 3 ft/min. How fast is the top of the ladder moving down when the bottom of the ladder is 8 feet from the wall?

(8) Let $f(x) = \sqrt[3]{x+1}$.

- **(a)** Find the linear approximation to $f(x)$ at 0.
- **(b)** Use this linear approximation to estimate: $\sqrt[3]{1.06}$
- **(c)** Use you calculator to compute $\sqrt[3]{1.06}$ correct to 5 decimal places.

(9) Use a linear approximation to estimate 2.01^4. (Hint: Use the linear approximation of x^4 at 2.)

(10) Compute the differential dy for the function $y = \sqrt{1+x^3}$

Five more questions. Show clearly all your working out and reasoning. Only do these questions when you are sure you understand the first ten.

(11) Compute: $\dfrac{d}{dx}\left(1+x^{12}\right)^7$

(12) The length of a rectangle is increasing at 6 cm/s. The rate of change of its width is -5 cm/s (in other words the width is decreasing at 5 cm/s). Find the rate of change of the area of the rectangle when the length is 12 cm and the width is 10 cm.

(13) Two cars are approaching an intersection, one from the south at 50 mi/h and the other from the east at 40 mi/h. What is the rate of change of the distance between the cars when they are both one mile from the intersection? (Hint: your answer should be negative.)

(14) Find the linear approximation to $\cos(x)$ at π.

(15) Use a linear approximation to estimate: $\dfrac{1}{5.03}$

Answers to questions (1)-(10):

(1) $x^5\sec^2 x + 5x^4\tan x$

(2) $\dfrac{x\sec^2 x - 4\tan x}{x^5}$

(3) $-12x^{11}\sin(x^{12})$

(4) Area is increasing at $100\ cm^2/s$

(5) $-4/9$

(6) The distance is increasing at $250\sqrt{3}\approx 433.013\ mi/h$

(7) The top is moving down at $4\ ft/min$.

(8) (a) $L(x)=1+\dfrac{x}{3}$, (b) 1.02, (c) 1.01961

(9) 16.32

(10) $dy=\dfrac{3}{2}(1+x^3)^{-1/2}x^2\,dx$