Math 31, Homework 4 on sections 2.5, 2.6, 2.7

Write all your working out and answers on your own notepaper - no need to write the questions. Please use lots of space.

It is very important that you show clearly any work you had to do to get your answers. Just writing the answer down with no work shown is usually not enough. Do all 15 questions - they are worth 2 points each. Hand in your solutions next week only.

For these first 10 questions, *check that your answers match the solutions on page 4*. If you don't get the same answer then go back and see where you went wrong.

(1) Define y as a function of x by

$$y = (3x + 8)^4$$

(a) Express y as a composition of functions $y = f(g(x))$ by finding the inside function $g(x)$ and the outside function $f(x)$.

(b) Now use the chain rule to compute $\dfrac{dy}{dx}$

(2) Use the chain rule to find: $\dfrac{d}{dx} \cos\left(\dfrac{1}{x^6}\right)$

(3) Compute $f'(x)$ for

$$f(x) = \left(\frac{x+1}{x-1}\right)^{100}$$

(4) Find the equation of the tangent line to the curve

$$y = \sqrt{1 + x^3} \qquad \text{at the point} \qquad (2, 3)$$

(5) Compute the derivative of: $\tan(\tan(x^9 + 1))$

(6) The curve

$$x^4 + 3x^2y^2 - y^3 = 5$$

implicitly defines y as a function of x. Find $\dfrac{dy}{dx}$

(7) Calculate $\dfrac{dy}{dt}$ by implicit differentiation if

$$\frac{t^2}{t+y} = y^2 + 1$$

(8) Find y' for the curve $\sin y + \cos x = \tan x$

(The notation y' means the first derivative $\frac{dy}{dx}$)

(9) A ball is thrown vertically up and its height after t seconds is

$$s(t) = 30t - 5t^2 \quad \text{meters.}$$

(a) Using the differentiation formulas, find the velocity of the ball: $v = s'$

(b) Find the acceleration of the ball: $a = v'$

(c) When does the ball have zero velocity and what does that mean?

(d) Find the maximum height of the ball.

(e) Find the velocity of the ball as it hits the ground.

(10) A tank has water draining out of it. If the volume V in gallons of water left in the tank after t minutes is

$$V = 2000\left(1 - \frac{t}{50}\right)^2$$

(a) Find the volume of water left after 40 minutes.

(b) At what rate is the water draining out of the tank at that time.

(c) When is the tank empty?

(Give the correct units for parts (a), (b) and (c).)

Five more questions. Show clearly all your working out and reasoning. Only do these questions when you are sure you understand the first ten.

(11) Use the chain rule to find the derivative of: $\sin^{99} x$

(Remember $\sin^n x$ means $(\sin x)^n$)

(12) Compute $f'(x)$ for

$$f(x) = \sqrt{x + \sqrt{x}}$$

(13) For the curve

$$x^3 + xy + y^3 = 11$$

(a) Find y'

(b) Find the slope of the tangent line at $(1, 2)$.

(14) Find y' for the curve

$$\sin(xy) = \cos(x + y)$$

(15) A ball is thrown vertically up and its height after t seconds is

$$s(t) = 48t - 4t^2 \quad \text{meters.}$$

(a) Using the differentiation formulas, find the velocity of the ball: $v = s'$

(b) When does the ball have zero velocity?

(c) Find the maximum height of the ball.

(d) Find the velocity of the ball as it hits the ground.

Answers to questions (1)-(10):

(1) (a) $g(x) = 3x + 8$ and $f(x) = x^4$ (b) $\dfrac{dy}{dx} = 12(3x+8)^3$

(2) $6x^{-7}\sin(x^{-6})$

(3)

$$f'(x) = -\frac{200}{(x-1)^2}\left(\frac{x+1}{x-1}\right)^{99}$$

(4) $y = 2x - 1$

(5)

$$\sec^2(\tan(x^9+1)) \cdot \sec^2(x^9+1) \cdot 9x^8$$

(6)

$$\frac{dy}{dx} = \frac{-4x^3 - 6xy^2}{6x^2y - 3y^2}$$

(7)

$$\frac{dy}{dt} = \frac{t^2 + 2ty}{t^2 + 2y(t+y)^2}$$

(8)

$$y' = \frac{\sin x + \sec^2 x}{\cos y}$$

(9) (a) $v = 30 - 10t$, (b) $a = -10$

(c) $v = 0$ when $t = 3$ so the ball has zero velocity at 3 seconds. Means the ball has stopped going up and starting to come down.

(d) 45m , (e) -30 m/s

(10) (a) 80 gallons left, (b) The water is draining out at 16 gallons per minute, (c) It takes 50 minutes to empty.