Math 31, Homework 3 on sections 2.3, 2.4

Do these 12 questions and *check that your answers match the solutions on page 2*. They will not be collected, but similar questions could appear on the next quiz and on the final. Let me know if you are having any difficulty doing them.

(1) Let

$$f(x) = 19.2x^{100} + 15^3$$

and use the differentiation formulas to find $f'(x)$.

(2) With the differentiation formulas, compute:

$$\frac{d}{dx}\left(3x^3 - \frac{5}{2}x^2 - 37 - x^{-4}\right)$$

(3) Find: $\dfrac{d}{dt}\left(5t + \sqrt{3}\right)^2$

(4) Use the quotient rule to calculate: $\dfrac{d}{dx}\dfrac{x^2+1}{x-2}$

(5) Suppose

$$f(3) = 1, \quad f'(3) = 2, \quad g(3) = 4, \quad g'(3) = -2.$$

Then compute

- **(a)** $(f+g)'(3)$ (Means evaluating the derivative of the sum $f(x) + g(x)$ at $x = 3$.)
- **(b)** $(fg)'(3)$
- **(c)** $(f/g)'(3)$
- **(d)** $(g/f)'(3)$

(6) Find all the points where the curve

$$y = 3x^4 - 16x^3 + 18x^2 + 6$$

has a horizontal tangent.

(7) Let $g(x) = (x^3+1)(x^2-1)$.

- **(a)** Find $g'(x)$ using the product rule for differentiation.
- **(b)** Find $g'(x)$ by first multiplying out the product and then differentiating.
- **(c)** Check that you get the same answer in (a) and (b).

(8) Use the product rule to compute: $\dfrac{d}{dt}\left(\sin t \cos t\right)$

(9) Let

$$f(\theta) = \frac{\sin\theta}{1+\cos\theta}$$

and find $f'(\theta)$

(10) What is the equation of the tangent line to $y = \tan x$ at the point $(0, 0)$.

(11) Let $h(x) = x\sin x$ and find $h^{(4)}(x)$. This is the fourth derivative.

(12) Compute these limits exactly:

$$(a) \quad \lim_{x\to 0}\frac{\sin 5x}{6x} \qquad (b) \quad \lim_{x\to 0}\frac{1-\cos x}{2\sin x} \qquad (c) \quad \lim_{x\to 0}\frac{x^2+5x}{\sin x}$$

Hint - use the limit laws and the results we saw in class:

$$\lim_{x\to 0}\frac{\sin x}{x} = 1, \qquad \lim_{x\to 0}\frac{\cos x - 1}{x} = 0$$

Answers to questions (1)-(12):

(1) $f'(x) = 1920x^{99}$

(2) $9x^2 - 5x + 4x^{-5}$

(3) $50t + 10\sqrt{3}$

(4) $\dfrac{x^2-4x-1}{(x-2)^2}$

(5) (a) 0 (b) 6 (c) $5/8$ (d) -10

(6) The curve has horizontal tangents at the three points: $(0, 6),\ (1, 11),\ (3, -21)$

(7) You should get $g'(x) = 5x^4 - 3x^2 + 2x$ either way.

(8) $\cos^2 t - \sin^2 t$ ($\cos^2 t$ means $(\cos t)^2$ and same for $\sin$)

(9) $\dfrac{1}{1+\cos\theta}$ (Hint: we used the identity $\cos^2\theta + \sin^2\theta = 1$)

(10) The equation of the tangent line is $y = x$

(11) $h^{(4)}(x) = x\sin x - 4\cos x$

(12) (a) $5/6$ (b) 0 (c) 5