## Math 31, Homework 2 on sections 1.8, 2.1, 2.2

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Write all your working out and answers on your own notepaper - no need to write the questions. Please use lots of space.

It is very important that you show clearly any work you had to do to get your answers. Just writing the answer down with no work shown is usually not enough. Do all 15 questions - they are worth 2 points each. Hand in your solutions next week only.

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For these first 10 questions, *check that your answers match the solutions on page 4*. If you don't get the same answer then go back and see where you went wrong.

**(1)** Let $f(x)$ be function defined by

$$f(x) = \begin{cases} 1-x & \text{if } x < 1 \\ x^2 - 1 & \text{if } x \geq 1. \end{cases}$$

**(a)** Sketch the graph of $f(x)$.

**(b)** Does $\lim_{x \to 1} f(x)$ exist?

**(c)** Explain if $f(x)$ is continuous or not at $x = 1$.

**(2)** The function

$$g(x) = x\sqrt{7 + x^2}$$

is continuous for all real numbers $x$. Use this fact to compute:

$$\lim_{x \to 3} g(x)$$

**(3)** **(a)** Let $f(x) = x^3 + 3x^2 + 3x - 2$. Is $f(x)$ continuous for all real numbers $x$? (Hint: we had a theorem about continuity of polynomials and rational functions.)

**(b)** Explain why the *Intermediate Value Theorem* shows that there is at least one solution to the equation

$$x^3 + 3x^2 + 3x - 2 = 0$$

in the interval $(0, 2)$.

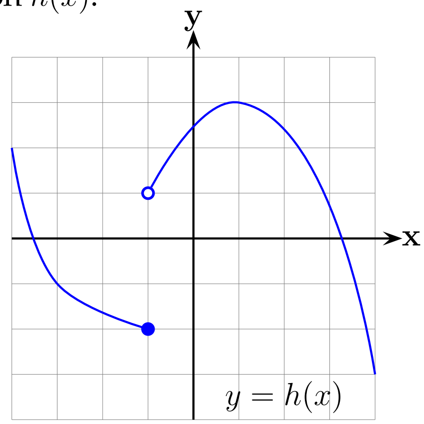
**(4)** Let $f(x) = x^2 - 5x$. Use the limit definition of derivative to calculate $f'(3)$. (No points if you use the differentiation formulas such as $\frac{d}{dx}x^n = nx^{n-1}$ here.)

**(5)** Use your answer from the last question to write the equation of the tangent line to the curve $y = x^2 - 5x$ at the point $P(3, -6)$.

**(6)** Sketch the graph of $g(x)$ if you know that:

$$g(0) = 1, \quad g'(0) = -1, \quad g'(2) = 0, \quad g'(3) = 0, \quad g'(4) = -3$$

**(7)** This is the graph of the function $h(x)$.




Estimate:

**(a)** $h(1)$

**(b)** $h'(1)$

**(c)** $h'(3)$

**(d)** $h'(-1)$

**(8)** Let $C(t)$ be the daily cost in dollars to heat a building when the outside temperature is $t$ degrees Fahrenheit.

**(a)** What does $C(58)$ mean?

**(b)** What does $C'(58)$ mean, and what are its units?

**(c)** Do you expect $C'(58)$ to be positive or negative? Explain.

**(9)** A ball is thrown vertically up and its height after $t$ seconds is

$$s(t) = 30t - 5t^2 \quad \text{feet.}$$

**(a)** Use the limit definition of derivative to find $s'(t)$

**(b)** Use the limit definition of derivative to find $s''(t)$ (the derivative of $s'(t)$).

**(c)** Now give the height, velocity and acceleration of the ball (in the correct units) at $t = 4$ seconds.

**(10)** Let $f(x) = \dfrac{1}{2+x}$

**(a)** Use the limit definition of derivative to find $\frac{d}{dx} f(x)$

**(b)** Is your formula for part (a) true for all real numbers $x$? Explain.

Do these five next questions when you are sure you understand the first ten. Show clearly all your working out and reasoning.

**(11)** Let $f(x)$ be function defined by

$$f(x) = \begin{cases} 1 - x & \text{if } x < 1 \\ x & \text{if } x \geq 1. \end{cases}$$

- **(a)** Sketch the graph of $f(x)$.
- **(b)** Does $\lim_{x\to 1} f(x)$ exist?
- **(c)** Explain if $f(x)$ is continuous or not at $x = 1$.

**(12)** Let $f(x) = x^4 - 3x + 1$. Explain why the *Intermediate Value Theorem* shows that there is at least one solution to the equation

$$x^4 - 3x + 1 = 0$$

in the interval $(-1, 1)$.

**(13)** Let $f(x) = 2 + \dfrac{1}{x}$. Use the limit definition of derivative to calculate $f'(1)$. Use this answer to write the equation of the tangent line to the curve $y = 2 + \dfrac{1}{x}$ at the point $P(1, 3)$.

**(14)** A ball is thrown vertically up on the moon and its height after $t$ seconds is

$$s(t) = 20t - t^2 \quad \text{feet.}$$

- **(a)** Use the limit definition of derivative to find $s'(t)$
- **(b)** Use the limit definition of derivative to find $s''(t)$ (the derivative of $s'(t)$).
- **(c)** Now give the height, velocity and acceleration of the ball (in the correct units) at $t = 4$ seconds.

**(15)** Let $f(x) = \sqrt{x+1}$

- **(a)** Use the limit definition of derivative to find $\frac{d}{dx} f(x)$
- **(b)** For which real numbers is your formula for part (a) true? Explain.

**Answers to questions (1)-(10):**

*(1)* (a) The graph looks line a line on the left and a parabola on the right - make a table of values and plot points. (b) The limit does exist and equals $0$. (c) $f(x)$ is continuous at $x = 1$ because $f(1) = 0$ and so

$$\lim_{x\to 1} f(x) = f(1)$$

In other words the graph of $f(x)$ near $x = 1$ does not have any gaps.

*(2)* Since $g(x)$ is continuous the limit equals $g(3) = 12$.

*(3)* (a) Yes. $f(x)$ is a polynomial and a theorem tells us that polynomials are always continuous at every real number $x$. (b) Check that $f(0)$ is negative and $f(2)$ is positive. Then the Intermediate Value Theorem tells us that there is a number $c$ between $0$ and $2$ so that $f(c) = 0$. $c$ is the solution.

*(4)* $f'(3) = 1$

*(5)* $y = x - 9$

*(6)* The graph crosses the $y$-axis at $1$, decreasing at a $45^o$ angle. It passes horizontally through $x = 2$ and $x = 3$ and then decreases again steeply.

*(7)* $(a)\quad h(1) = 3, \qquad (b)\quad h'(1) = 0,$

$(c)\quad h'(3) = -4$ approximately, $\qquad (b)\quad h'(-1)$ does not exist.

*(8)* (a) $C(58)$ is the cost in dollars to heat the building when it's $58$ degrees outside. (b) $C'(58)$ is the rate of change of dollar cost with respect to degree at $58$ degrees. So, if the temperature changes by a small amount from $58$ degrees, then the heating cost will change by $C'(58)$ times this amount. Its units are dollars per degree. (c) You expect $C'(58)$ to be negative. If the outside temp goes from $58$ to $59$ then the heating cost should go down (a negative change).

*(9)* $(a)\quad s'(t) = 30 - 10t, \qquad (b)\quad s''(t) = -10$

(c) So at $4$ seconds the ball has height $s(4) = 40$ feet, velocity $s'(4) = -10$ ft/sec (negative means height is decreasing, so ball is coming down) and acceleration $s''(4) = -10$ ft/sec$^2$ (feet per second squared - negative acceleration comes from gravity giving a downward force).

*(10)*

$$\frac{d}{dx} f(x) = f'(x) = \frac{-1}{(2+x)^2}$$

$f(x)$ and its derivative $f'(x)$ make sense for all real numbers except $-2$. In other words, they both have domain $(-\infty, -2) \cup (-2, \infty)$