Math 31, Extra Credit Homework 10 on sections 4.3, 4.4, 4.5

Write all your working out and answers on your own notepaper - no need to write the questions. Please use lots of space.

It is very important that you show clearly any work you had to do to get your answers. Just writing the answer down with no work shown is usually not enough. Do all 15 questions - they are worth 2 points each. Hand in your solutions next week only.

For these first 10 questions, *check that your answers match the solutions on page 3*. If you don't get the same answer then go back and see where you went wrong.

(1) Use part 1 of the Fundamental Theorem of Calculus to find:

$$\frac{d}{dx}\int_3^x \tan^2(3t)\,dt$$

(2) Find: $\displaystyle \frac{d}{dx}\int_0^{x^3} \sqrt{2t+1}\,dt$

(3) Use part 2 of the Fundamental Theorem of Calculus to compute:

$$\int_1^4 \frac{2+x^2}{\sqrt{x}}\,dx$$

(4) Calculate the indefinite integral: $\displaystyle \int 12x^5\,dx$

(5) Find: $\displaystyle \int \left(t - 13\sin(t) + \sqrt{2}\right)\,dt$

(6) Suppose a particle moves in a straight line with velocity $v(t) = 1/2 + \sin(t)$ cm/s.

- **(a)** Is the particle always moving forward?
- **(b)** What is the displacement of the particle between $t = 0$ and $t = 5\pi$ seconds? (In other words, what is the net change of its position?)

(7) If water is leaking out of a tank at a rate of $f(t)$ gallons per minute at time t, what does $\displaystyle \int_0^{120} f(t)\,dt$ represent?

(8) Use the substitution rule to evaluate: $\displaystyle \int \sec^2(7x+\pi)\,dx$

Differentiate your answer and check you get $\sec^2(7x+\pi)$.

(9) Find: $\displaystyle\int \frac{\cos(\sqrt{t})}{\sqrt{t}}\,dt$

(10) Compute the definite integral:

$$\int_{2/3}^{1} (3x-2)^{1000}\,dx$$

Five more questions. Show clearly all your working out and reasoning. Only do these questions when you are sure you understand the first ten.

(11) Find: $\displaystyle\frac{d}{dx}\int_{x}^{10} \sec(t^3)\,dt$ (Hint: recall that $\int_b^a f(t)\,dt = -\int_a^b f(t)\,dt$.)

(12) Evaluate: $\displaystyle\int x^3\sqrt{x^2+1}\,dx$

(13) Compute: $\displaystyle\int_{\sqrt{\pi}}^{\sqrt{2\pi}} 8x\sin(x^2)\,dx$

(14) Find the indefinite integral: $\displaystyle\int \tan^2(\theta)\sec^2(\theta)\,d\theta$ (Hint: $u=\tan(\theta)$.)

(15) Suppose $f(x)$ is an odd function. This means that $f(-x) = -f(x)$. Use the following steps to prove that

$$\int_{-a}^{a} f(x)\,dx = 0 \quad \text{for all} \quad a \geq 0.$$

(a) Write $\displaystyle\int_{-a}^{a} f(x)\,dx = \int_{-a}^{0} f(x)\,dx + \int_{0}^{a} f(x)\,dx$. Why is this true?

(b) Use the substitution $u=-x$ in the first integral on the right to show:

$$\int_{-a}^{0} f(x)\,dx = -\int_{a}^{0} f(-u)\,du = \int_{0}^{a} f(-x)\,dx$$

(c) Therefore we have shown

$$\int_{-a}^{a} f(x)\,dx = \int_{-a}^{0} f(x)\,dx + \int_{0}^{a} f(x)\,dx = \int_{0}^{a} f(-x)\,dx + \int_{0}^{a} f(x)\,dx.$$

What is the final step to see this is zero?

Answers to questions (1)-(10):

(1) $\tan^2(3x)$

(2) $3x^2\sqrt{2x^3+1}$

(3) $82/5$

(4) $2x^6 + C$

(5) $t^2/2 + 13\cos(t) + \sqrt{2}t + C$

(6) (a) No, the particle is sometimes moving backwards (ie has negative velocity).

(b) Displacement is $2 + 5\pi/2$ cm.

(7) The integral represents the number of gallons of water that have leaked out of the tank in the first two hours.

(8) $\dfrac{1}{7}\tan(7x+\pi) + C$

(9) $2\sin(\sqrt{t}) + C$

(10) $1/3003$