## Math 31, Homework 1 on sections 1.4, 1.5, 1.6

Write all your working out and answers on your own notepaper - no need to write the questions. Please use lots of space.

It is very important that you show clearly any work you had to do to get your answers. Just writing the answer down with no work shown is usually not enough. Do all 15 questions - they are worth 2 points each. Hand in your solutions next week only.

For these first 10 questions, check that your answers match the solutions on page 4 . If you don't get the same answer then go back and see where you went wrong.
(1) The point $P(1,1)$ is on the curve $y=x^{3}$.
(a) Find the slope of the secant line through $Q(2,8)$ and $P(1,1)$.
(b) Find the slope of the secant line through $R\left(1.1,1.1^{3}\right)$ and $P(1,1)$.
(c) Find the slope of the secant line through $S\left(1.001,1.001^{3}\right)$ and $P(1,1)$.
(d) Use the above work to estimate the slope of the tangent line to the curve $y=x^{3}$ at $P(1,1)$. (Hint: what number are your answers from parts (a), (b) and (c) getting close to?)
(2) A phone is accidentally dropped out a window. Physics tells us that the distance it has fallen after $t$ seconds is $s(t)=4.9 t^{2}$ feet. (We are calling the position function $s(t)$.)
(a) How far has the phone fallen after 1 second?
(b) Find the average velocity (in feet per second) of the phone between $t=1$ second and $t=2$ seconds.
(c) Find the average velocity of the phone between $t=1.99$ second and $t=2$ seconds.
(d) Use the above work to estimate the instantaneous velocity of the phone at $t=2$ seconds.
(e) Convert the instantaneous velocity at $t=2$ seconds into miles per hour.
(3) Explain in your own words what this means:

$$
\lim _{x \rightarrow 3} f(x)=10
$$

(4) We want to estimate numerically

$$
\lim _{x \rightarrow 1} \frac{2^{x}-2}{x-1} .
$$

( $2^{x}$ means 2 to the power $x$.) Do this by evaluating $\left(2^{x}-2\right) /(x-1)$ on your calculator at $x=0.9, x=0.99, x=0.9999, x=1.1, x=1.01$ and $x=1.0001$. What is your estimate?
(5) Numerically estimate

$$
\lim _{x \rightarrow 0^{+}} x^{x}
$$

(6) This is the graph of the function $h(x)$.


Find the following:
(a) $h(-1)$
(b) $\lim _{x \rightarrow-1^{-}} h(x) \quad$ (left-sided limit)
(c) $\lim _{x \rightarrow-1^{+}} h(x) \quad$ (right-sided limit)
(d) $\lim _{x \rightarrow-1} h(x) \quad$ (two-sided limit)
(7) Suppose

$$
\lim _{x \rightarrow 0} f(x)=3, \quad \lim _{x \rightarrow 0} g(x)=2, \quad \lim _{x \rightarrow 2} f(x)=-1, \quad \lim _{x \rightarrow 2} g(x)=0 .
$$

Use the limit laws to calculate
(a) $\lim _{x \rightarrow 0}\left((f(x))^{2}+4 g(x)\right)$
(b) $\lim _{x \rightarrow 2} \frac{g(x)+4}{f(x)-3}$
(8) Find

$$
\lim _{x \rightarrow 3}\left(x^{2}-x+2\right)
$$

using the limit laws. Say which law you are using at each step.
(9) Compute these limits using algebra and the limit laws:
(a) $\lim _{h \rightarrow 0} \frac{(h+4)^{2}-16}{4 h}$
(b) $\lim _{x \rightarrow-2} \frac{\frac{1}{x}+\frac{1}{2}}{x+2}$
(10) Compute this limit using algebra and the limit laws:

$$
\lim _{x \rightarrow 2} \frac{x^{2}+x-6}{x-2}
$$

When you have understood the first ten questions then try these five further questions. Show clearly all your working out and reasoning.
(11) A skydiver jumps out of a plane. The distance they have fallen after $t$ seconds is $s(t)=4.9 t^{2}$ feet.
(a) How far has the skydiver fallen after 20 seconds?
(b) Find the average velocity of the skydiver between $t=19.99$ seconds and $t=20$ seconds.
(c) Use the above work to estimate the instantaneous velocity at $t=20$ seconds.
(d) Convert the instantaneous velocity at $t=20$ seconds into miles per hour. (The physics formula $s(t)=4.9 t^{2}$ is really for falling without air resistance.)
(12) Explain in your own words what this means (don't use the word limit in your answer):

$$
\lim _{x \rightarrow 4^{-}} f(x)=\infty
$$

(13) Numerically estimate

$$
\lim _{x \rightarrow 0^{+}} x^{1 / x}
$$

(Hint: $x^{1 / x}$ means $x$ to the power $1 / x$. Try $x=0.1$ first.)
(14) Suppose

$$
\lim _{x \rightarrow 0} f(x)=3, \quad \lim _{x \rightarrow 0} g(x)=16, \quad \lim _{x \rightarrow 2} f(x)=0, \quad \lim _{x \rightarrow 2} g(x)=7
$$

Use the limit laws to calculate

$$
\lim _{x \rightarrow 0}\left((f(x))^{3}-\sqrt{g(x)}\right)
$$

(15) Compute this limit using algebra and the limit laws:

$$
\lim _{x \rightarrow 5} \frac{x^{2}-7 x+10}{x-5}
$$

## Answers to questions (1)-(10):

(1) (a) 7 ,
(b) 3.31,
(c) 3.003,
(d) 3 .
(2) (a) 4.9 ft ,
(b) $14.7 \mathrm{ft} / \mathrm{sec}$,
(c) $19.551 \mathrm{ft} / \mathrm{sec}$,
(d) about $19.6 \mathrm{ft} / \mathrm{sec}$,
(e) 13.36 miles/hour
(3) It means that as $x$ get closer and closer to 3 from both sides then $f(x)$ gets closer and closer to 10 .
(4) Make a table of these six values. Your estimate should be 1.386 .
(5) You should estimate that

$$
\lim _{x \rightarrow 0^{+}} x^{x}=1
$$

(6) (a) $h(-1)=-2$,
(b) $\lim _{x \rightarrow-1^{-}} h(x)=-2$,
(c) $\lim _{x \rightarrow-1^{+}} h(x)=1$,
(d) $\lim _{x \rightarrow-1} h(x)$ does not exist.
(7) $(a) \quad 17, \quad(b)-1$.
(8) On the first step use the difference and sum laws. Answer is 8 .
(9) $(a) \quad 2, \quad(b) \quad-1 / 4$.
(10) 5

