Math 30, Homework 5 on sections 2.6, 2.7

Write all your working out and answers on your own notepaper - no need to write the questions. Please use lots of space.

It is very important that you show clearly any work you had to do to get your answers. Just writing the answer down with no work shown is usually not enough. Do all 15 questions - they are worth 2 points each. Hand in your solutions next week only.

For these first 10 questions, *check that your answers match the solutions on page 4*. If you don't get the same answers then look at your notes or the book or ask me. Only do the last five questions when you are sure you understand the first ten.

(1) Find the domains of these rational functions:

(a) $\dfrac{x+3}{x^2-49}$

(b) $\dfrac{x-3}{x^2+49}$

(2) Let $f(x) = -1/x$ and complete these statements:

(a) As $x \to -\infty$, $f(x) \to$ what?

(b) As $x \to \infty$, $f(x) \to$ what?

(c) As $x \to 0^-$, $f(x) \to$ what?

(d) As $x \to 0^+$, $f(x) \to$ what?

(3) Find the x-intercepts and vertical asymptotes of the graph of the rational function

$$g(x) = \frac{(x+2)(x+4)}{(x+1)(x-2)}.$$

(4) Find the x-intercepts and vertical asymptotes of the graph of the function

$$h(x) = \frac{2x+1}{2x^2+x-1}.$$

(5) Find the horizontal asymptotes of the graphs of these functions if they have one:

(a) $p(x) = \dfrac{-3x+1}{x-2}$

(b) $q(x) = \dfrac{x}{4x^2+9}$

(c) $r(x) = \dfrac{x^2}{5}$

(6) Let
$$f(x) = \frac{3x}{x-2}.$$

Find the x and y intercepts of the graph of this rational function. Find its vertical and horizontal asymptotes. Plot some extra points and then sketch the graph. Remember to label and number the axes.

(7) Let
$$g(x) = \frac{(x+2)(x+4)}{(x+1)(x-2)}$$

as in question 3. You already found the x-intercepts and vertical asymptotes. Find the y intercept and horizontal asymptote. Plot some extra points and then sketch the graph. Remember to label and number the axes.

(8) Solve
$$(x+1)(x-7) \geq 0.$$

Graph the solution set on the number line and also give the solutions in interval notation.

(9) Solve
$$\frac{x+2}{x+1} < 0.$$

Graph the solution set on the number line and also give the solutions in interval notation.

(10) Solve and give the solutions in interval notation
$$\frac{(x+2)(x+4)}{(x+1)(x-2)} < 0.$$

Five more questions. Show clearly all your working out and reasoning.

(11) Let $f(x) = \dfrac{x+1}{x-1}$ and complete these statements:

    (a) As $x \to -\infty$, $f(x) \to$ what?

    (b) As $x \to \infty$, $f(x) \to$ what?

    (c) As $x \to 1^-$, $f(x) \to$ what?

    (d) As $x \to 1^+$, $f(x) \to$ what?

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(12) Find the horizontal asymptotes of the graphs of these functions if they have one:

(a) $p(x) = \dfrac{x^2 + 1}{x - 2}$

(b) $q(x) = \dfrac{x}{x - 2}$

(c) $r(x) = \dfrac{1}{x - 2}$

(13) Let

$$k(x) = \frac{2x^2 - 2}{x^2 - 9}.$$

Find the x and y intercepts of the graph of this rational function. Find its vertical and horizontal asymptotes. Plot some extra points and then sketch the graph. Remember to label and number the axes.

(14) Solve

$$x^2 - x - 6 \leq 0.$$

Graph the solution set on the number line and also give the solutions in interval notation.

(15) Solve and give the solutions in interval notation

$$\frac{(x + 3)(x - 4)}{(x + 1)^2} > 0.$$

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Answers to questions (1)-(10):

(1) (a) Domain is all real numbers except -7 and 7, (b) Domain is all real numbers.

(2) *(a)* As $x \to -\infty$, $f(x) \to 0$

   (b) As $x \to \infty$, $f(x) \to 0$

   (c) As $x \to 0^-$, $f(x) \to \infty$

   (d) As $x \to 0^+$, $f(x) \to -\infty$

(3) The x-intercepts of $g(x)$ are the points $(-2, 0)$ and $(-4, 0)$. The vertical asymptotes are the lines $x = -1$ and $x = 2$.

(4) The x-intercept of $h(x)$ is the point $(-1/2, 0)$. The vertical asymptotes are the lines $x = -1$ and $x = 1/2$.

(5) (a) The horizontal asymptote for $p(x)$ is the line $y = -3$, (b) The horizontal asymptote for $q(x)$ is the line $y = 0$, (c) $r(x)$ does not have a horizontal asymptote.

(6) The graph of $f(x)$ crosses the axes only at $(0, 0)$. The vertical asymptote is at $x = 2$ and the horizontal asymptote is at $y = 3$. Use $f(-1) = 1$, $f(1) = -3$, $f(3) = 9$ and $f(4) = 6$ to plot four more points on the graph and then draw the graph approaching the asymptotes.

(7) The graph of $g(x)$ has x-intercepts at $(-4, 0)$ and $(-2, 0)$. It crosses the y axis at $(0, -4)$. The vertical asymptotes are at $x = -1$, $x = 2$ and the horizontal asymptote is at $y = 1$. Use $g(-3) = -0.1$, $g(3) = 8.75$ and $g(4) = 4.8$ to plot three more points on the graph and then draw the graph approaching the asymptotes.

(8) In interval notation the solution set is $(-\infty, -1] \cup [7, \infty)$. On the number line, shade to the left of -1 and the right of 7. Put filled in circles at -1 and 7.

(9) In interval notation the solution set is $(-2, -1)$. Shade the corresponding parts of the number line and use open circles for the boundary points since they are not solutions.

(10) The solution set is $(-4, -2) \cup (-1, 2)$.

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