

## Math 30, Extra Credit Homework 10 on section 5.5

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Write all your working out and answers on your own notepaper - no need to write the questions. Please use lots of space.

It is very important that you show clearly any work you had to do to get your answers. Just writing the answer down with no work shown is usually not enough. Do all 15 questions - they are worth 2 points each. Hand in your solutions next week only.

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For these first 10 questions, *check that your answers match the solutions on page 2*. If you don't get the same answers then go back and see where you went wrong. Only do the last five questions when you are sure you understand the first ten.

- (1) Does the equation  $\sin(x) = 2$  have any solutions? Explain.
- (2) Check if  $x = \pi/4$  is a solution to  $\cos(x) = 1/2$ .
- (3) Find the solutions to  $\sin(x) = 1$  for  $x$  in the interval  $[0, 2\pi)$ . (Hint: where does the graph of  $y = \sin(x)$  meet the graph of the horizontal line  $y = 1$ ?)
- (4) Next, list all solutions to  $\sin(x) = 1$  by using the fact that sine is periodic:  $\sin(x+2\pi) = \sin(x)$
- (5) Find the two solutions to  $\sin(x) = \sqrt{2}/2$  for  $x$  in the interval  $[0, 2\pi)$ . Then list all solutions. (Hint: where does the graph of  $y = \sin(x)$  meet the graph of the horizontal line  $y = \sqrt{2}/2$ ? The first solution is a special angle and you can find the second by symmetry.)
- (6) Find all solutions to:  $3 \cos(x) - 1 = \cos(x)$   
(Hint: first solve for  $\cos(x)$ .)
- (7) Find all solutions to:  $\cos(2x) = 1/2$
- (8) Using your result from the previous question, find all solutions to  $\cos(2x) = 1/2$  that have  $x$  in the interval  $[0, 2\pi)$ .
- (9) Find all solutions to:  $\sin(x) = -\sqrt{3}/2$
- (10) Find all solutions to  $\sin(3x) = -\sqrt{3}/2$  that have  $x$  in the interval  $[0, 2\pi)$ .

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Five more questions. Show clearly all your working out and reasoning.

- (11) Does the equation  $\cos(x) = -11/10$  have any solutions? Explain.
- (12) List all solutions to:  $\sin(x) = -1$
- (13) Find all solutions to:  $5 \sin(x) + \sqrt{2} = 3 \sin(x)$
- (14) Find all solutions to:  $\sin(2x) = 1/2$
- (15) Find all solutions to  $\sin(2x) = 1/2$  that have  $x$  in the interval  $[0, 2\pi)$ .

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**Answers to questions (1)-(10):**

- (1) The range of  $\sin(x)$  is  $[-1, 1]$  so  $\sin(x) = 2$  has no solutions.
- (2) We have  $\cos(\pi/4) = \sqrt{2}/2$ , so  $x = \pi/4$  is not a solution to  $\cos(x) = 1/2$ .
- (3) The solution is  $\pi/2$ .
- (4) All solutions are given by  $\pi/2 + 2\pi n$  where  $n$  is any integer.
- (5) The solutions  $\pi/4$  and  $\pi - \pi/4 = 3\pi/4$  are in  $[0, 2\pi)$ . So all solutions are:

$$\frac{\pi}{4} + 2\pi n, \quad \frac{3\pi}{4} + 2\pi n \quad \text{for } n \text{ any integer.}$$

- (6) The equation simplified to  $\cos(x) = 1/2$  and the solutions are:

$$\frac{\pi}{3} + 2\pi n, \quad \frac{5\pi}{3} + 2\pi n \quad \text{for } n \text{ any integer.}$$

- (7) The solutions are:  $\frac{\pi}{6} + \pi n, \quad \frac{5\pi}{6} + \pi n$  for  $n$  any integer.

- (8) Four of the previous solutions are in the interval  $[0, 2\pi)$  (when  $n = 0, 1$ ):

$$\frac{\pi}{6}, \quad \frac{7\pi}{6}, \quad \frac{5\pi}{6}, \quad \frac{11\pi}{6}$$

- (9) The solutions are:  $\frac{4\pi}{3} + 2\pi n, \quad \frac{5\pi}{3} + 2\pi n$  for  $n$  any integer.

- (10) There are six solutions in the interval  $[0, 2\pi)$  :

$$\frac{4\pi}{9}, \quad \frac{10\pi}{9}, \quad \frac{16\pi}{9}, \quad \frac{5\pi}{9}, \quad \frac{11\pi}{9}, \quad \frac{17\pi}{9}$$