

ALGEBRA I. PROBLEM SET 3.

These problems will not be graded. Make sure you can do all 10 and let me know if you have questions about them. Check through all the other exercises in Chapter 4 as well.

- #1 [Q10, §4.1.] This question defines and develops properties of *double cosets*. See the text.
- #2 [Q4, §4.2.] Use the left regular representation of Q_8 to produce two elements of S_8 which generate a subgroup of S_8 that is isomorphic to Q_8 .
- #3 [Q7(b), §4.2.] Show that Q_8 is not isomorphic to a subgroup of S_7 .
- #4 [Q2, §4.3.] Find all conjugacy classes and their sizes in the following groups:
 - (a) D_8 ,
 - (b) Q_8 ,
 - (c) A_4 .
- #5 [Q13, §4.3.] Find all finite groups with exactly 2 conjugacy classes.
- #6 [Q4, §4.5.] Exhibit all Sylow 2-subgroups and Sylow 3-subgroups of D_{12} and $S_3 \times S_3$.
- #7 [Q7, §4.5.] Exhibit all Sylow 2-subgroups of S_4 and find elements of S_4 which conjugate one of these into each of the others.
- #8 [Q13, §4.5.] Prove that a group of order 56 has a normal Sylow p -subgroup for some prime p dividing its order.
- #9 [Q24, §4.5.] Prove that if G is a group of order 231 then the following are true.
 - (a) $Z(G)$ contains a Sylow 11-subgroup of G .
 - (b) A Sylow 7-subgroup is normal in G .
- #10 [Q2, §4.6.] Find all normal subgroups of S_n for every $n \geq 5$.