Please note: You should fully justify your answers.

1 Finding equations of lines

1. Find an equation of the line that:
   (a) has slope $-2$ and $y$–intercept 11. $y = -2x + 11$
   (b) has slope $-\frac{5}{2}$ and $y$–intercept 0. $y = -\frac{5}{2}x$
   (c) has slope $\frac{3}{4}$ and passes through the point $(0, -4)$. $y = \frac{3}{4}x - 4$
   (d) has the same slope as $2y - 4x = 10$ and the same $y$–intercept as $y = 5x - 3$. $y = 2x - 3$
   (e) has slope $-5$ and passes through the point $(-2, 3)$. $y = -5x + 7$
   (f) has slope 0 and passes through the point $(3, 5)$. $y = 5$
   (g) is vertical and passes through the point $(-3, 0)$. $x = -3$
   (h) passes through the points $(-5, 13)$ and $(1, -5)$. $y = -3x - 2$
   (i) passes through the points $(-2, 4)$ and $(1, 7)$. $y = x + 6$
   (j) passes through the points $(3, 0)$ and $(6, 2)$. $y = \frac{2}{3}x - 2$
   (k) passes through the points $(-1, 5)$ and $(-1, -3)$. $x = -1$
   (l) passes through $(0, 0)$ and $(3, -5)$. $y = -\frac{5}{3}x$
   (m) passes through the points $(2, 4)$ and $(-3, 4)$. $y = 4$
   (n) passes through the points $(0, 4)$ and $(-5, 0)$. $y = \frac{4}{5}x + 4$
   (o) passes through the points $\left(\frac{2}{3}, -\frac{1}{9}\right)$, and $\left(-\frac{15}{2}, -\frac{6}{5}\right)$. $y = \frac{2}{15}x - \frac{1}{5}$
   (p) has the same slope as $3x - 5y = -2$ and the same $x$–intercept as $-2x - 3y = 6$. $y = \frac{3}{5}x - 3$
   (q) has the same $x$–intercept as $-2x + 3y = -2$ and the same $y$–intercept as $x - y = 3$. $y = -3x - 3$

2. Find the equations for each of the lines in Figure 1.

2 Parallel lines, Perpendicular lines

1. For each of the following pairs of lines, decide whether they are parallel, perpendicular or neither.
   (a) $y = 3x - 4, y = -3x + 2$ Neither
   (b) $y = \frac{2}{3}x, y = -\frac{3}{2}x + 9$ Perpendicular
   (c) $2x - 3y = 7, 2x - 3y = 5$ Parallel
   (d) $3x + y = -2, -2x + 3y = 0$ Neither
   (e) $-5x + 2y = 8, 2x + 5y = -3$ Perpendicular
   (f) $y = 3x + 8, 3x + y = -3$ Neither
Figure 1: The lines of Question 2

(a) \( y = -2x + 3 \)

(b) \( y = x - 2 \)

(c) \( y = x \)

(d) \( y = -x \)

(e) \( y = 3x - 4 \)

(f) \( y = \frac{2}{3}x - 2 \)
(g) \( y = 2x - 7, \ y = 2x + 9 \quad \text{Parallel} \\
(h) \ y = 5x - 7, \ y = -\frac{x}{5} + 9 \quad \text{Perpendicular} \\
(i) \ 2x + 3y - 9 = 0, \ y = -\frac{2x}{3} - 2 \quad \text{Parallel} \\

2. Find an equation for the line that:
   (a) passes through \((-1, 3)\) and is parallel to the line \(y = 3x - 5\). \[ y = 3x + 6 \]
   (b) is parallel to \(2x - 5y = 6\) and passes through \((1, -2)\). \[ 2x - 5y = 12 \]
   (c) is parallel to \(x = -3\) and passes through \((5, 9)\). \[ x = 5 \]
   (d) is perpendicular to \(x = 2\) and passes through \((3, 4)\). \[ y = 4 \]
   (e) is perpendicular to \(y = -\frac{x + 2}{3}\) and passes through \((0, -2)\). \[ 3x + 2y = 13 \]
   (f) passes through the point \((3, 2)\) and is perpendicular to \(2x - 3y = 5\). \[ 2x - 3y = 5 \]
   (g) has the same \(y\)-intercept as \(3x - 4y = 8\) and is parallel to \(y = -5x + 11\). \[ y = -5x - 2 \]

3. Verify that the following four points are the corners of a parallelogram.
   \[ P(-4, -9), \ Q(-2, -3), \ R(-4, -7), \ S(-6, -13) \]

   \textit{Answer.} \(RQ\) and \(PS\) each have slope 2, so they are parallel. Also \(PQ\) and \(RS\) are parallel because each have slope 3. So, \(PQRS\) is a parallelogram. \(\square\)

4. Verify that the following three points are the corners of a right triangle.
   \[ A(2, 4), \ B(0, 0), \ C(4, 3) \]

   \textit{Answer.} The slope of \(AB\) is \(m_1 = 2\) and the slope of \(AC\) is \(m_2 = -\frac{1}{2}\). Since \(m_1m_2 = -1\) it follows that \(AB\) and \(AC\) are perpendicular, so the angle \(A\) of \(ABC\) is a right angle. Therefore \(ABC\) is a right triangle. \(\square\)

5. Verify that the following four points are the corners of a rectangle.
   \[ A(1, 1), \ B(4, 4), \ C(-1, 3), \ D(2, 6) \]

   \textit{Answer.} By computing the slopes we see that \(AC\) and \(BD\) are parallel and so are \(AB\) and \(CD\). Additionally, \(AC\) is perpendicular to \(AB\). So \(ABCD\) is a parallelogram with a right angle. So it has to be a rectangle. \(\square\)

6. Consider again a line \(l\) with equation in standard form
   \[ Ax + By + C = 0 \]

   where \(A, B, C\) are real numbers and at least one of \(A, B\) is non-zero.

   (a) Prove that a line with equation
   \[ Ax + By + D = 0 \]

   where \(D\) is any number, is parallel to \(l\).

   (b) Prove that a line with equation
   \[ Bx - Ay + D = 0 \]

   where \(D\) is any number, is perpendicular to \(l\).