1. Perform the indicated operations and simplify:

   (a) \[ \frac{n^2 - 3n - 10}{n^2 + n - 2} \]
   (b) \[ \frac{1 + \frac{5}{3y}}{ \frac{6}{2x} + \frac{5}{y}} \]
   (c) \[ \frac{2x^2 - x}{2x^2 + x - 1} \]

   (d) \[ \left( \frac{x^2 - 2xy - 3y^2}{x^2 - 9y^2} \right) \left( \frac{2x^2 - xy}{2x^2 + xy - y^2} \right) \]

   (e) \[ \frac{9ab}{3a + b} = \frac{a^2 - 6a}{a^2 - 4a - 12} \]

   (f) \[ \frac{10}{x^2 - 5x} + \frac{2}{x} \]

   (g) \[ \frac{3}{n^2 - 5n - 36} + \frac{2}{n^2 + 3n - 4} \]

   (h) \[ \frac{1}{x - 3} - \frac{3}{3 - x} \]

   (i) \[ \frac{1}{6x(x - 2)^2} - \frac{1}{4x^2(x - 2)} \]

2. Solve:

   (a) \[ \frac{1}{2x - 7} + \frac{x - 5}{4x^2 - 49} = \frac{4}{6x - 21} \]

   (b) \[ \frac{4}{5y - 3} = \frac{2}{3y + 7} \]

   (c) \[ n + \frac{1}{n} = \frac{53}{14} \]

   (d) \[ 2 + \frac{4}{x - 2} = \frac{8}{x^2 - 2x} \]

3. Simplify (express results with positive exponents only and rationalized denominators):

   (a) \[ 16^{-3/4} \]

   (b) \[ (4x^5y^{-2})^{-2} \]

   (c) \[ \frac{4\sqrt{12}}{\sqrt{5}} \]

   (d) \[ \sqrt[3]{\frac{3}{8}} \]

   (e) \[ \sqrt[3]{\frac{27}{64}} \]

   (f) \[ \sqrt[4]{81x^4y^4z^7} \]

   (g) \[ (-8x^{-6}y^{12})^{1/3} \]

4. Perform the indicated operations and simplify:

   (a) \[ 3\sqrt{20} - \sqrt{5} - 3\sqrt{15} \]

   (b) \[ \sqrt{3} + 2 \left( \sqrt{24} \right) - 6 \left( \sqrt{81} \right) \]

   (c) \[ -4\sqrt{2} \left( 2\sqrt{5} - 3\sqrt{6} \right) \]

   (d) \[ \left( 2\sqrt{6} + 3\sqrt{5} \right) \left( \sqrt{8} - 3\sqrt{12} \right) \]

   (e) \[ \frac{\sqrt{7}}{3\sqrt{2} - 5} \]

   (f) \[ \frac{\sqrt{12}}{4} + \frac{\sqrt{27}}{6} \]
5. Solve for $x$ and check your solutions:

(a) $\sqrt{2x-5} = 5$  
(b) $\sqrt{4x+2} = \sqrt{3x+4}$  
(c) $4\sqrt{x} + 5 = x$  
(d) $\sqrt{x+1} - \sqrt{2x} = 1$

6. (a) Solve by factoring: $x^2 - 7x + 12 = 0$
(b) Solve with the quadratic formula: $4x^2 + 2x + 1 = 0$
(c) Solve by completing the square: $2x^2 - 8x - 3 = 0$
(d) Solve by any method: $x^2 + 10x + 26 = 0$

7. Simplify (express your results in the form $a + bi$ for $a, b$ real):

(a) $(4 - 8i) - (8 - 3i)$  
(b) $(\sqrt{-4}) (\sqrt{-16})$  
(c) $7i (-9 + 3i)$
(d) $(10 + 2i)(-2 - i)$  
(e) $\frac{-1 - 3i}{4 - 5i}$  
(f) $2 (\cos 120^\circ + i \sin 120^\circ)$

8. Given $f(x) = 3x^2 - 2$, determine:

(a) $f(2)$  
(b) $f(-3)$  
(c) $f(1/2)$  
(d) $f(a)$

9. Sketch the graphs of each of the given functions, indicating the $x$ and $y$ intercepts, the vertex, the axis of symmetry and stating the maximum or minimum value of the function:

(a) $f(x) = x^2 - 3$  
(b) $g(x) = -2x^2 + 2$  
(c) $h(x) = -(x + 2)^2$
(d) $k(x) = -(x + 2)^2 + 2$  
(e) $w(x) = x^2 + x + 1$

10. Given $f(x) = \frac{x^2 - 9}{x^2 + 2x - 15}$

(a) Determine the values of $x$ for which the function is defined
(b) Evaluate: $f(0)$
(c) Evaluate: $f(-3)$

11. Graph:

(a) $2x + 3y \leq 6$  
(b) $x - 2y < 4$

12. Given the functions $f(x) = 2x + 1$ and $g(x) = \frac{1}{2}x - \frac{1}{2}$ sketch both graphs on the same set of axes.

13. Sketch each pair of functions on the same set of axes:

(a) $f(x) = 3^x$ and $g(x) = 3^{-x}$  
(b) $f(x) = 3^x$ and $g(x) = \log_3 x$

14. Solve for $x$ (use the definitions and properties of exponents and logarithms):

(a) $3^{x-1} = 81$  
(b) $7^{-x} = 49$  
(c) $25^x = 125$
(d) $\log_4 1 = x$  
(e) $\log_6 (6^{-8}) = x$  
(f) $\log_2 x = -4$
15. Solve and write the answer in set notation:
   (a) $|2x - 1| = 3$  (b) $|x| - 1 = 4$  (c) $|2x - 1| \leq 3$  (d) $|x| > 5$  (e) $|2 - 3x| \geq 1$

16. Find the center and radius for each circle:
   (a) $x^2 + y^2 - 4y = 12$  (b) $x^2 + y^2 = 9$  (c) $x^2 - 6x + y^2 - 4y = 8$

17. (a) Find $\theta$, to the nearest whole degree, if $\sin \theta = 0.1234$ and $\cos \theta < 0$
   (b) Find $\sin \theta$ if $\cos \theta = -3/5$ and $\theta$ is in Quadrant III.

18. Without using a calculator, determine the exact value of:
   (a) $\sin^2(30^\circ) + \cos^2(30^\circ)$  (b) $\tan(60^\circ)$  (c) $\sec^2(45^\circ) - \tan^2(45^\circ)$  (d) $\sin(\pi/6) - \cos(\pi/4)$

19. If each of the following points $P$ are on the terminal side of angle $\theta$ in standard position with $0 \leq \theta < 360^\circ$, draw $\theta$ and determine the value of the six trigonometric functions of $\theta$:
   (a) $P = (3, -2)$  (b) $P = (-3, 4)$  (c) $P = (2, 4)$

20. For each of the following angles $\theta$, draw them in standard position, choose a specific point on the terminal side of $\theta$ and determine the exact values of $\sin \theta$, $\cos \theta$ and $\tan \theta$ without using a calculator:
   (a) $\theta = \frac{5\pi}{6}$  (b) $\theta = 315^\circ$  (c) $\theta = 270^\circ$

21. Evaluate $\sin \theta$, $\cos \theta$ and $\tan \theta$ exactly for each of the following angles:
   (a) $\theta = 210^\circ$  (b) $\theta = -240^\circ$  (c) $\theta = 675^\circ$

22. Solve the following (clearly specify the unknown, draw a labeled diagram if appropriate and state the solution in words):
   (a) The time a person takes to paddle a kayak 2 miles downstream is the same as the time to paddle half a mile upstream. If the rate of the current is 3 mph, what is the person’s paddling rate in still water?
   (b) Bill is standing on top of a 175 foot cliff overlooking a lake. The measure of the angle of depression to a boat is $29^\circ$. How far is the boat from the bottom of the cliff (rounded to one decimal place)?
   (c) Suppose that the height in meters of a golf ball, hit from a tee, is approximated by $y = -5t^2 + 10t + 15$ where $t$ is the time in seconds. Find the maximum height of the ball and the time it reaches this maximum height.

23. Graph each equation for $-2\pi \leq x \leq 2\pi$:
   (a) $y = 2\sin x$  (b) $y = 3\cos x$  (c) $y = -\cos x$

24. A central angle of $\theta = 30^\circ$ is contained in a circle with radius $r = 30$ inches. Find (leaving all results in terms of $\pi$):
   (a) the length of the arc subtended by $\theta$
   (b) the area of the sector determined by $\theta$
25. Find the exact values of the area and perimeter of this right triangle:

![Diagram of a right triangle with sides labeled and angle measures]

26. Verify the following trigonometric identities:
   (a) \( \csc \theta \tan \theta \cos \theta = 1 \)
   (b) \( \csc \theta - \sin \theta = \cot \theta \cos \theta \)
   (c) \( \tan^2 \theta + 1 = \sec^2 \theta \)
   (d) \( \cos^2 \theta - \sin^2 \theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \)

27. (a) A hot-air balloon rises vertically. An observer stands on level ground at a distance of 125 feet from a point on the ground directly below the passenger’s compartment. How far, to the nearest foot, does the balloon rise if the angle of elevation changes from 20° to 30°?

   (b) A state trooper is hiding 30 feet from a straight highway with a speed limit of 65 mph. One second after a truck passes, the angle \( \theta \) between the highway and the line of observation from the patrol car to the truck is measured.

      (i) If \( \theta = 15° \) does the truck driver get a speeding ticket (1 mile = 5,280 ft)?
      (ii) If \( \theta = 30° \) does the truck driver get a speeding ticket?

Answers

1. (a) \( \frac{n - 5}{n - 1} \) (b) \( \frac{3xy + 5x}{3(3y + 5x)} \) (c) \( \frac{x}{x + 1} \) (d) \( \frac{x}{x + 3y} \) (e) \( \frac{9ab + 18b}{3a + b} \)
   (f) \( \frac{2}{x - 5} \) (g) \( \frac{n + 15}{(n - 9)(n + 4)(n - 1)} \) (h) \( \frac{4}{x - 3} \) (i) \( \frac{x + 6}{12x^2(x - 2)^2} \)

2. (a) \( x = 22 \) (b) \( y = -17 \) (c) \( n = \frac{2}{7}, \frac{7}{2} \) (d) \( x = -2 \)

3. (a) \( \frac{1}{8} \) (b) \( \frac{y^4}{16x^{16}} \) (c) \( \frac{8\sqrt{15}}{5} \) (d) \( \frac{\sqrt{5}}{4} \) (e) \( \frac{3}{4} \) (f) \( 3xyz^2 \sqrt{3yz} \) (g) \( \frac{-2y^4}{x^2} \)

4. (a) \( -4\sqrt{5} \) (b) \( -13\sqrt{3} \) (c) \( -8\sqrt{10} + 24\sqrt{3} \)
   (d) \( 8\sqrt{3} - 36\sqrt{2} + 6\sqrt{10} - 18\sqrt{15} \) (e) \( -\frac{3\sqrt{14} + 5\sqrt{7}}{7} \) (f) \( \sqrt{3} \)
5. (a) $x = 15$  (b) $x = 2$  (c) $x = 25$  (1 is an extraneous solution)  (d) $x = 0$  (8 is an extraneous solution)

6. (a) $x = 3, 4$  (b) $x = \frac{-1 \pm i\sqrt{3}}{4}$  (c) $x = \frac{2 \pm \sqrt{22}}{2}$  (d) $x = -5 \pm i$

7. (a) $-4 - 5i$  (b) $-8$  (c) $-21 - 63i$  (d) $-18 - 14i$  (e) $\frac{11}{41} - \frac{17}{41}i$  (f) $-1 \pm i\sqrt{3}$

8. (a) $f(2) = 10$  (b) $f(-3) = 25$  (c) $f(1/2) = -5/4 = -1\frac{1}{4}$  (d) $f(a) = 3a^2 - 2$

9. (a) Axis of symmetry $x = 0$
   Vertex $(0, -3)$
   Minimum $y = -3$
   $y$-intercept $(0, -3)$
   $x$-intercepts $(-\sqrt{3}, 0), (\sqrt{3}, 0)$

(b) Axis of symmetry $x = 0$
   Vertex $(0, 2)$
   Maximum $y = 2$
   $y$-intercept $(0, 2)$
   $x$-intercepts $(-1, 0), (1, 0)$

(c) Axis of symmetry $x = -2$
   Vertex $(-2, 0)$
   Maximum $y = 0$
   $y$-intercept $(0, -4)$
   $x$-intercept $(-2, 0)$

(d) Axis of symmetry $x = -2$
   Vertex $(-2, 2)$
   Maximum $y = 2$
   $y$-intercept $(0, -2)$
   $x$-intercepts $(-2 \pm \sqrt{2}, 0)$

(e) Axis of symmetry $x = -1/2$
   Vertex $(-1/2, 3/4)$
   Minimum $y = 3/4$
   $y$-intercept $(0, 1)$
   $x$-intercepts none

10. (a) $(-\infty, -5) \cup (-5, 3) \cup (3, \infty)$ or all real numbers except $-5$ and $3$  (b) $f(0) = 3/5$  (c) $f(-3) = 0$
11.

(a) ![Graph of a line]

(b) ![Graph of an increasing line]

12.

![Graph with functions f(x) and g(x)]

13.

(a) ![Graph of functions f(x) and g(x)]

(b) ![Graph of functions f(x) and g(x)]

14.

(a) \( x = 5 \)  \hspace{1cm}  (b) \( x = -2 \)  \hspace{1cm} (c) \( x = \frac{3}{2} \)  \hspace{1cm} (d) \( x = 0 \)  \hspace{1cm} (e) \( x = -8 \)  \hspace{1cm} (f) \( x = \frac{1}{16} \)
15. (a) $\{-1, 2\}$  (b) $\{-5, 5\}$  (c) $\{x \mid -1 \leq x \leq 2\}$  (d) $\{x \mid x < -5 \text{ or } x > 5\}$  (e) $\{x \mid x \leq 1/3 \text{ or } x \geq 1\}$

16. (a) center $(0, 2)$ radius 4  (b) center $(0, 0)$ radius 3  (c) center $(3, 2)$ radius $\sqrt{21}$

17. (a) $\theta = 173^\circ$  (b) $\sin \theta = -4/5$

18. (a) 1  (b) $\sqrt{3}$  (c) 1  (d) $1/2 - \sqrt{2}/2 = (1 - \sqrt{2})/2$

19. (a) $\sin \theta = -2/\sqrt{13} = -2\sqrt{13}/13$

(b) (−3, 4)  (c) (2, 4)

\[
\begin{align*}
\sin \theta &= -2/\sqrt{13} = -2\sqrt{13}/13 \\
\cos \theta &= 3/\sqrt{13} = 3\sqrt{13}/13 \\
\tan \theta &= -2/3 \\
\csc \theta &= -\sqrt{13}/2 \\
\sec \theta &= \sqrt{13}/3 \\
\cot \theta &= 3/2
\end{align*}
\]

\[
\begin{align*}
\sin \theta &= 4/5 \\
\cos \theta &= -3/5 \\
\tan \theta &= 4/3 \\
\csc \theta &= 5/4 \\
\sec \theta &= 5/3 \\
\cot \theta &= 3/4
\end{align*}
\]

\[
\begin{align*}
\sin \theta &= 4/(2\sqrt{5}) = 2\sqrt{5}/5 \\
\cos \theta &= 2/(2\sqrt{5}) = \sqrt{5}/5 \\
\tan \theta &= 4/2 = 2 \\
\csc \theta &= \sqrt{5}/2 \\
\sec \theta &= \sqrt{5}/3 \\
\cot \theta &= 1/2
\end{align*}
\]
20. (a) 
\[
\begin{align*}
\sin \frac{5}{6} \pi &= \frac{1}{2} \\
\cos \frac{5}{6} \pi &= -\frac{\sqrt{3}}{2} \\
\tan \frac{5}{6} \pi &= \frac{1}{-\sqrt{3}} = -\frac{\sqrt{3}}{3}
\end{align*}
\]

(b) 
\[
\begin{align*}
\sin 315^\circ &= -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2} \\
\cos 315^\circ &= \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \\
\tan 315^\circ &= -1 = -1
\end{align*}
\]

(c) 
\[
\begin{align*}
\sin 270^\circ &= -\frac{1}{1} = -1 \\
\cos 270^\circ &= 0 = 0 \\
\tan 270^\circ &= -\frac{1}{0}, \text{ undefined}
\end{align*}
\]

21. 
(a) \( \sin 210^\circ = -\sin 30^\circ = -1/2 \), \( \cos 210^\circ = -\cos 30^\circ = -\sqrt{3}/2 \), \( \tan 210^\circ = \tan 30^\circ = \sqrt{3}/3 \)
(b) \( \sin(-240^\circ) = \sin 60^\circ = \sqrt{3}/2 \), \( \cos(-240^\circ) = -\cos 60^\circ = -1/2 \), \( \tan(-240^\circ) = -\tan 60^\circ = -\sqrt{3} \)
(c) \( \sin 675^\circ = -\sin 45^\circ = -\sqrt{2}/2 \), \( \cos 675^\circ = \cos 45^\circ = \sqrt{2}/2 \), \( \tan 675^\circ = -\tan 45^\circ = -1 \)

22. 
(a) The person’s paddling rate in still water is 5 mph.
(b) The boat is 315.7 feet from the bottom of the cliff.
(c) The maximum height of the ball is 20 meters which it reaches in 1 second.

23. 
\[
\begin{align*}
\sin 5\frac{\pi}{6} &= \frac{1}{2} \\
\cos 5\frac{\pi}{6} &= -\frac{\sqrt{3}}{2} \\
\tan 5\frac{\pi}{6} &= \frac{1}{-\sqrt{3}} = -\frac{\sqrt{3}}{3}
\end{align*}
\]
24.  
(a) arc length = $5\pi$ inches  
(b) area = $75\pi$ square inches

25.  
\[ \text{Area} = 2 \tan 30^\circ \text{ in}^2 = \frac{2\sqrt{3}}{3} \text{ in}^2; \text{ perimeter} = 2\sqrt{3} + 2 \text{ in} \]

26.  
To prove these identities, use algebra and the basic identities
\[
\begin{align*}
\tan \theta &= \frac{\sin \theta}{\cos \theta} \\
\cos^2 \theta + \sin^2 \theta &= 1 \\
csc \theta &= \frac{1}{\sin \theta} \\
\sec \theta &= \frac{1}{\cos \theta} \\
cot \theta &= \frac{1}{\tan \theta}
\end{align*}
\]

27.  
(a) The balloon rises 27 feet.

(b) (i) The truck is traveling at 111.96 ft/sec which is 76.34 mph and the driver gets a ticket. (ii) The truck is traveling at 51.96 ft/sec which is 35.43 mph and the driver does not get a speeding ticket.