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1 Greetings

I am very happy today. I did not know that so many people loved me enough to gather at Queens College to wish me a healthy, long, and productive life over and above the 71 years I have already lived. It includes my teacher Shlomo Sternberg, present here on skype, and my "almost"-teachers Hyman Bass, and Cliff Earle. Alex Lubotzky came from Israel, Ulrich Pinkall from Germany, and Shiga from Japan. If I have counted correctly there are 14 people among the speakers who are above 65, and 5 below 65, of which only 3 in their 30s to 50s. There are many more in the audience who are in their 50s and below. I interpret this as: we old people have done something right. And of course that something right, is that we have done mathematics.

The conference of this type is new for the Math department at Queens College, although it had many distinguished mathematicians like Arthur Sard, Leo Zippin, Banesh Hoffman, Edwin Moise, ... before, on its faculty.

I find this Conference especially gratifying since I already went back to India in 2001, enjoyed several leaves without pay, and finally retired from Queens College, in Feb 2008. However I keep coming back to Queens college and Graduate Center twice a year and enjoy my emeritus positions with all the office and library/computer advantages.

For a long time, I felt that people here thought that I was an Indian in America. I was even more surprised that the people in India thought that I was an American in India. But today I feel that the people from both the continents think of me as their own.

One sign of it is that I may tell you that for 70 years I never ran for any office like that of HoD. But in India, some mathematicians nominated me for the office of the President of the Ramanujan Mathematical Society. In April 2013, I was elected with 75 percent of the voters approving me!

I wish to use this position to invite many of you to India to develop more networks, and light many more lamps of higher mathematics in India.

To begin with, I must thank my many colleagues at Queens College: President Muyskens, Dean Liebovich, the HoD Prof. Wallace Goldberg, our untiring Math secretary Minnie, who has been like my elder sister taking care of me in many ways for the last 28 years. Then there is Valerie, and her whole family, her parents, husband, and her five children. Valerie gave me something which I consider as the highest honor I have received in my life. She and her husband Yuri gave their first son the first name Valerie's father's name, Lawrence, and the second name, my name, Ravi. So here is the most handsome, fairest, blue-eyed Lawrence Ravi I know in the world. He is thinking of a career in science.

For this Conference, I especially wish to thank Sudeb Mitra, Yunping Jiang, Zhe Wang, and Blendi Koroveshi, who put in days of efforts to arrange this Conference on Conformal Geometry and Riemann Surfaces. I have to spend at least a year to learn the high-tech they have used to make this Conference a success. I have worked in other areas, but these subjects have been the inspiration of my life.

We may not quite realize it in our day-to-day lives, but we are truly living in momentous times. Millennia have been squeezed into centuries, and centuries have been squeezed into decades. It is not yet 100 years since Ramanujan, a clerk working at an obscure place in Chennai, India, wrote to Hardy, a distinguished professor of mathematics at a world-renowned university in UK. He was invited by Hardy, who made arrangement for his travel and living expenses by way of a scholarship. One great, unexpected, result in Mathematics came from their fruitful collaboration. But far more than that it significantly added to the human consciousness about Mathematics and the sciences all over the world. Can you believe, all this happened less in less than 100 years, that is less than 40 thousand days?

Unprecedented, great changes in our attitudes and ways of thinking are taking place every few years in our short lives. We all know, that Knowledge is Power, Knowledge is Money, Knowledge is Prestige, and all that, but far more than that, Knowledge remains the most ancient, and supreme human value.

In this transformation US has played, is playing, and is inspiring the whole world to play a great historic role.

Let me add a nuance to the romanticization of Ramanujan's story. I think, even Ramanujan, just a 100 years ago, probably, could not have been offered a position at a university in UK without breaking its written or unwritten statutes. But US broke down all such barriers. Harvard University supported me for 5 years as a student, and one year as a Research Fellow. Following that the Johns Hopkins University, Columbia University, Rutgers University, Indiana University, and finally the Queens College of the City University of New York offered me long-term positions, the last two, full professorships. In

between, IHES, France, University of Bonn, Germany, Institute for Advanced Study in Princeton, Institut Mittag Leffler in Sweden, U of Helsinki in Finland, Princeton U, U of Michigan, U of Colorado, offered me long peaceful stays, where I could work. Following a recommendation of my adviser Shlomo Sternberg, Prof. Marcel Berger arranged a position for me in Paris, France, the land of Poincaré. Shortly after that Prof. Klingenberg opened the doors of the land of Riemann, Hurwitz, and Weyl for me. These are my heroes in Mathematics. Then I came in contact with John Milnor, Michael Atiyah, Armand Borel, and S. S. Chern, and then my friends Dennis Sullivan, Bill Thurston, Misha Gromov, and Shing -Tung Yau. I really do not know how to count my blessings!

On this occasion, I do remember my loving parents, my first teachers. These are intensely personal feelings, and not of much interest to others. Let me only say that I do not owe my interest in Mathematics to my parents, but whatever else that you may have found good in me, I owe it entirely to my parents. I should share with you one thing: my father, who was a Gandhian in thinking, taught me before I finished my school, the Upanishads, and Bhagavad-Geeta, the Old and New Testaments, and the Koran. This reading had a great impact on my thinking. In fact, when I try to analyze my subconscious mind, I feel that why I was attracted to Shlomo Sternberg as my thesis adviser was not just differential geometry, but also Shlomo's scriptural scholarship, which many of us greatly admire!

I would like to use this opportunity to say something which you may also like to hear

i) Thoughts on Mathematics at the age of 70+.
ii) Thoughts about the Mathematics profession.
iii) My Teachers.
iv) A philosophical note.

2 Thoughts on Mathematics

I have summarized my thoughts on Mathematics in the following picture: A Mathematician's View of the Tree of Knowledge.

You know the Tree of Knowledge is a part of the Old Testament. It also occurs in the 15th chapter of the Bhagavad-Geeta. For our purposes, let us simply consider it as "Arts and Sciences". In Galileo's and Newton's times, what we now call natural sciences was a part of a single subject: "Natural Philosophy". In the picture, which is admittedly a mathematician's perspective, I have put the natural sciences below Mathematics, and emerging upwards from Philosophy. Downwards from Philosophy are the social sciences, and the Arts —- this is just a mathematician's view, not a judgment on their significance.

You know, Mathematics is not a popular subject. But Music is. And Mathematics and Music are so closely related. Why should Mathematics be not as

popular as Music? I think, that we should admit that our communication of mathematics is not up to the mark. We should consciously try to change this popular reaction to Mathematics. In our communication of Mathematics to intelligent laymen, or to our colleagues and students at a college or a university, before heading into a particular area, we should try to explain the overall logical structure of Mathematics and its larger role in science and society. The "logical structure" includes not only the deductive part we always present in our class-rooms, but also the inductive part, following the steps from the original experience to theory-building, which will potentially lead to new discoveries.

In particular, we should tell our students and interested people outside the profession, how we ourselves think in Mathematics. We think in Mathematics in terms of Pictures and Numbers. They symbolize the two categories, Space and Number, of mathematical thought. At the High School these develop into Plane Euclidean Geometry, Combinatorics, and School Algebra. At the College level these develop further into 2-dimensional synthetic geometries, Calculus, and so on. At the master's level and beyond these develop further into several disciplines which we separately recognize today.

Until the 18th century, existence of mathematical entities, their "ontology", was mainly based on sense-perception, the same as in Physics, and other natural sciences. But at the end of the 19th century, Dedekind and Cantor ushered us into Set Theory. Now we follow 2-valued Logic, and express all mathematics in terms of sets, structures on sets, and maps among sets with structures. (We need "categories" in order to avoid Russell's paradox about the set of all sets. But I won't go into this aspect.) This has clarified our expression enormously. There is no mystery about "space-time". To begin with, locally, it is just 4-tuples of real numbers. "Velocity", "Mass", "Time", "Force" are not new, a priori, concepts in mathematics. "Mass" and "Time" are modelled on real numbers. (By the way, the "Real Line and Real Numbers", based on the Dedekind's notion of completeness of ordered sets, is one of the mysteries of mathematics, which, in my opinion, we cleverly hush up!) The "Velocity", "Force" are some sections of some fiber bundles and so on.

Hilbert described Set Theory as the paradise created by Cantor. I think, it is at least for the following three reasons.

First, it gave us the "definition" of infinity, and we came up with a startling conclusion that there are infinity of infinities — a view with which most traditional philosophers are still catching up.

Secondly, it changed the "ontology" of Mathematics. It helped us to get beyond the cruder pre-Cantor ontology based on sense-perception. For a mathematical entity to exist, we do not need to see it with our physical eyes, or touch it, or taste it, or smell it. It exists, so long as it can be constructed out of "known" sets using elementary axioms such as taking products, quotients, and maps among them. There is no issue that we shall never be able to see the real projective plane with our physical eyes. It exists as a mathematical entity as a quotient space of the 2-sphere in 3-space.

Thirdly, a great contribution of Set Theory is that it also gave us a "definition" of "Symmetry". The idea of "Symmetry" goes much beyond Mathematics. But with Set Theory in their repertoire, the mathematicians could make it precise in terms of groups (which is a set with an extra structure) and the actions of groups on sets. Verily, the classification of simple, connected, Lie groups, and the classification of finite simple groups, are the two great contributions of our fathers' generation and our generation. Hermann Weyl even advocated "Symmetry" as the third category of mathematical thought, besides "Space" and "Number".

So in the Tree-picture, I have put "Symmetry" vertically on the right, "looking" at all Mathematics at its left. At the "origin", the intersection of the horizontal and the vertical lines, I have put the "2-valued Logic, Set Theory, and Category Theory". That is how the mathematical community has agreed to *express* all its experience of all Mathematics.

This expression is extremely powerful, and is useful for all natural or social sciences. It is also concise, and has helped us to formulate large parts of mathematics in a deductive way, fulfilling Euclid's dream. In fact, set theory is really an extension of what Euclid called "General Notions". I have explained above that we do not need to introduce "velocity", "force", "mass", "time" as new undefined concepts. For theory-building, they can all be formulated once we accept "Real Line and Real Numbers" as "known" sets.

Let no one tell us "Math is not a science. It is merely a "Language" of science". Mathematics *is* a science. Only that we allow our "data" to be mental. Our data need not have a material model. For example, we find that some of the primes like $2, 5, 13, 17, \ldots$ are sums of two squares, whereas the primes $3, 7, 11, \ldots$ are not. This is our data. This data does not become a serious "Theorem" until Euler proved that the primes 2 and those congruent to 1, mod 4, are exactly the primes which are sums of two squares. Those congruent to 3, mod 4, are not. So the statement now applies to all primes, which are infinitely many.

Thus Mathematics is not only a Science, but it is a Science which gives ways to formulate theories for all physical and social sciences. *So the theory-building in all sciences is fundamentally mathematical in nature.*

So the sets, structures on sets, and maps among the sets with structures have provided an extremely powerful expression for the experience of mathematics. But is that experience equal to its expression? On this point, I may differ from some of you here. I can only give an analogy. I find the experience of mathematics is like the experience of seeing the Himalayan peaks, or the Atlantic ocean. In my opinion, Mathematics is deeper than its set-theoretic expression. It has to deal with the "Who Am I"-rectangle at the top of the Tree-picture. I shall return back to this point at the end.

But in practical terms, while teaching or communicating Mathematics, or while writing papers, let us be more generous to our potential audience. Let us make the subject more human. Let us communicate it with some semi-historical

perspective. Not so much the biographies of eminent mathematicians, although that too, to explain the social background in which even the eminent mathematicians work. But more emphasis on how the mathematical ideas developed over time.

Let us explain our subject with pictures which explain equations, and equations which explain pictures, juxtaposing the "space" and "number" categories of mathematical thought. Let our students thoroughly know that mathematics is not just numbers. In fact if you think about it, most mathematical entities are often expressible in terms of numbers, but they themselves are not numbers. For example, a vector in an n-dimensional vector space is expressible as an n-tuple of numbers, after choosing a basis, but intrinsically it is not that! An operator and a quadratic form on an n-dimensional vector space may both be expressed by an $n \times n$-matrix, after choosing a basis, but they are different entities, and they are different from the representing matrix. You can hardly call, a connection on a fiber bundle, a number.

Our big sense of pride lies in using Set Theory and 2-valued Logic, and present large parts of mathematics in a deductive way. But human mind comprehends inductive reasoning more easily. From particular examples, often provable in a simpler way, to a general theory. Also even one schematic picture is worth a thousand words, which have to be written carefully in conformity with the dictates of Set Theory.

3 Thoughts on the Mathematics Profession

Let us be happy that Mathematics is a noble profession. All societies acknowledge its importance, and allow us to make a respectable living. Let us not be too much pre-occupied about writing papers and getting them published, at least after we have gained tenures! Let us also try to spread the message of Mathematics.

Since Galileo's and Newton's times the progress in mathematics has been extremely rapid. Just note that not even 150,000 days have passed since Galileo's times. So sooner or later, perhaps already in our life-times a plateau must be reached for the pace of new discoveries in Mathematics. But even when the plateau will be reached, we shall have an enormous task of digesting what is discovered, and communicating the whole edifice to the larger community. That will be our recurring, all-time, commitment.

In this regard, India offered me better opportunities. The authorities were happy that I was interested in teaching. So along with some of my friends, I recommended to the National Board of Higher Mathematics (NBHM) in India to start an Advanced Training In Mathematics (ATM)-Program, in 2005. It has taken root, and is developing very well. Its management has been recently given to the National Center of Mathematics (NCM), which is a joint venture of the Tata Institute of Fundamental Research (TIFR), and the Indian Institute

of Technology in Mumbai, (IIT(Bombay)). Over 200 research mathematicians participate in it. It is aimed at post-master's doctoral students, and encourages them to gain an in-depth knowledge of 2 or 3 different fields before zeroing in on a thesis topic. NCM will soon have its own building situated in the IIT(Bombay) campus.

The Presidency of the Ramanujan Mathematical Society offers me a good opportunity to extend this philosophy to undergraduate and high school education. Professors Kumaresan and Ambat Vijaykumar of RMS have already been working in these areas for some time. Following the example of AMS, we intend to have many more regional meetings and many regional innovative Undergraduate Faculty Workshops. At the undergraduate level, we are interpreting Mathematics broadly, wiping out the distinction between Pure and Applicable Mathematics, to include Engineering, Statistics, and Computer Science.

I invite all of you to come to India, enjoy the different culture, natural scenery, new foods, and also exchange notes and contribute to the development of Mathematics in many ways. Love of Mathematics unites us all!

Writing a paper and getting it published has its plus and minus points. It need not be the only way we can enjoy and contribute to Mathematics. In fact this singular focus has made us self-centered, and unduly competitive. I know some of my colleagues are so competitive, that they compete not only with their colleagues, but they compete with their own students. Many of them would not like their students to consult with their colleagues.

In India, in many universities, there are rules, which, with my experience in the freer atmosphere in the West, I find very strange. In many Indian universities, after a student gets his masters, if he wants to go for a Ph.D., he needs to get attached to an adviser. Worse, he needs his adviser's certification for financial support every six months. The adviser often suggests some narrow research problem without providing its place in the broader mathematical landscape. He may discourage the student consulting with other mathematicians. Effectively, the adviser's limitations continue to remain the student's limitations for a long time. This hampers the progress of mathematics and the whole fun of doing mathematics is lost. That was partly the reason why the authorities in India agreed to start the ATM-activities mentioned above.

But these are complex issues. Some of them can be handled by devising alternate structures and reward systems. But to make changes in the established structures is usually very hard.

At the root, it really depends on the senior mathematicians recognizing that they should encourage a "co-operative" model over a "competitive" model of doing research. As much as all scientific research is finally a social activity, a "largely co-operative" and "moderately competitive" — just to ensure enough incentives — are the models which are, in my opinion, justifiable.

Considering the responsibility of the senior people in our profession, brings me to the consideration of the "priority issues" which have plagued our mathe-

matical community, all the time. Just compare: Mahavir was a contemporary of Buddha. Both developed largely similar meditational techniques, still existing and practised today. But they never indulged in any priority battles! In fact, Buddha reputedly said: "if you "follow" Buddha, then you are not a buddhist! Just take whatever that suits you best." By contrast, in the scientific community, Newton and Leibniz had priority battles over the discovery of Calculus. And following that, the English mathematicians did not adopt Leibniz's superior notation for a century. The Bernoulli brothers had a priority battle. I even do not recall what the battle was about. In my opinion, Gauss acted meanly towards Lobachevsky and Bolyai and denying them the credit of discovery of Hyperbolic Geometry. He wanted all the credits to himself, for the accounts, he would not publish in his life-time. If we ask Buddha —- or for that matter the larger world scientific community over the centuries —- about priority issues, Buddha would smile, shrug his shoulders, and say that these issues are far from the real issues of life. They are not worth a discussion. Instead, if you run into a problem, just concentrate on the breathing!

As a community, we need to develop a more mature view of the priority issues. As the group activity in Mathematics, is slowly increasing, I think, we shall develop such a view in the 21st century. Let the senior people among us consciously encourage group activity. Let us start talking to each other, crossing the boundaries of our specializations. Let us obliterate the difference between "Pure" and "Applicable" Mathematics. Let us appreciate different ways of thinking, looking at the same thing in different ways. That way, we shall increase the enjoyment of Mathematics.

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Finally, I would like to express my concern over something very important, although some of you may consider that this is not the occasion to raise it. But since you have been listening to me, let me use the occasion.

I am concerned over the role big money has started to play in our social life. This raises a more fundamental philosophical issue: what should be our attitude towards acquisition of money? We, in the scientific community, are all very intelligent people. How we think, and how we express, is likely to have some impact. So we should express freely on these matters.

My comments are motivated by the following life-experiences.

i) When I came to US, we could get a slice of pizza for 15 cents. Now it costs 2.25 dollars, if you are lucky! About 15-fold inflation. Our salaries have increased much less. We are all wondering, how has it come to pass?

ii) More seriously, President Eisenhower talked about the power of the Military-Industrial Complex in the 50s, and our Grothendieck protested against getting research-funding from the same establishment in the 60s. I remember one of my teachers, Prof. George Mackey, refused to accept NSF-grants.

iii) In our own times, Perelman refused to get the Fields Medal and the million-dollars prize. Some people called him a "nut". But I think, he was

trying to say something more fundamental. He should elaborate it some time. I request his friends to get an answer from him to this question. Perhaps he wanted to suggest that the acquisition of knowledge is a supreme human value. But mere acquisition of money can never be a human value.

iv) My concern is mainly coming from the rampant corruption in India in all walks of life, and the nexus between the politicians and the big corporations. The cost of education, and the cost of health-care are sky-rocketing, and a lot of corruption in these areas also. For example, the medical professionals are resorting to "cut-practice". In many colleges run by politicians, the "un-tenured" faculty have to sign a statement that they are getting X amount of salary, and actually they get much less.

v) At the same time, I find it disturbing that many college students, waste their precious mornings, in reading the financial pages of newspapers to check how their investments are faring, or getting excited by the stories on the internet how some kid hit a fortune by some lucky investment.

vi) Just see how our language has changed in the last 50 years: these days we don't even "earn" money, we "make" money!

vii) A quote by my good friend, Sylvain Cappell: A mathematician can make a lot of money — after he leaves mathematics.

viii) A quote by my good friend Todd Drum: In America, you can pay God to get anything. Praying will get you nowhere.

ix) In my childhood, "speculation" was a bad word. Now it has become a very respectable word. The economists tell us that in a capitalist society, private industry needs capital, and industry creates jobs. So a citizen who has some money to spare, is encouraged to speculate. The argument given is: we have to do this, in order to keep up with the inflation, and the inflation is indeed very real. But is creating jobs the primary motivation of the people who are asking us to speculate? They seem to be just after making more and more money. There is no transparency in much of this financial talk. I get a feeling that an "insider" information is necessary if you wish to make big money.

I am recalling these experiences keeping in view only a limited aim: what should be the attitude towards acquisition of money in our scientific community?

Most of us have come from a lower middle-class background. We often thought in our childhood, "if only I had money, I would do this or that for the world!". What percentage of us have kept that idealism?

I think, it was about the age of 5, that I learnt that Knowledge (with a capital K) is important. That Knowledge (with a capital K) is symbolized by a Goddess, we call her Saraswati, who looks like an elementary school teacher. She plays music. There are books around her. That Knowledge is supposed to make you free, and give peace and happiness. It is hard to acquire. It is like walking on a razor's edge. Mathematics is a part of that Knowledge. The scriptural literature describes Mathematics as the highest communicable part of that Knowledge. To acquire it, what we really need is the intense desire, sincerity, character, and a good teacher. If we have intense desire, the good

teacher may even come in our dream, and we get the instruction that is right for the stage we are in. But otherwise the Knowledge that makes one free, is absolutely free, like sun-shine and moon-light!

But that is definitely not the knowledge of financial markets. It does not make you free. It does not make you happy. It is like the chains of gold or platinum, but chains nevertheless. It binds you, and gives you tension, headaches, backaches, and sleepless nights. And it is not free. It always comes with a cost, which is picked up by the teams of brokers who sell the financial packages!

I am not saying anything new. I know, many of us are privately worrying about these deep issues of what should be the values we should promote in a society. I know, many of my colleagues in Queens College, are doing exemplary community service. Their children, students, and colleagues are inspired by these acts. I only wish, that we from the scientific community at large, should express our opinions more freely on these issues. We are very intelligent, but timid, people. We do not have the courage of a Gandhi. But collectively, we can make a difference.

I would like to mention here Mahatma Gandhi's thoughts on such issues, which unfortunately are not well-known even in India among the intellectual circles. However, thanks to Google, they are now at least readily available. Just Google: "Mahatma Gandhi, on Trustee-ship".

With a strong basis in the scriptural literature, and the "wisdom"-traditions of the world, Gandhi advocates the idea of the attitude of "Trustee-ship". Suppose you are super-intelligent. Then Gandhi recommends that a super-intelligent individual should consciously develop an attitude that "I don't have a personal claim on my intelligence. I am a Trustee. As a principle, I should keep my needs few, and after they are fulfilled, I should make my intelligence freely available for the benefit, first of my family, then of my community, and then of the society at large."

I think, if a majority of us consciously develop such attitude, that would solve the "priority" concerns in the scientific community — just as in the last forty years, we solved the problem of smoking in public spaces. It started in the US. The world followed.

Gandhi was thinking not in the context of super-intelligence, but in the context of super-wealth. Just read it from the original. I recommend it. It is interesting that Gandhi described "the Doctrine of Trusteeship is as complex as Euclid's concept of geometric point." Both are mental, non-material, ideas.

4 My Teachers

• Prof.s D. W. Kerkar, M. R. Railkar, and Mrs Apte (my undergraduate teachers)
• Prof.s Shlomo Sternberg, Raoul Bott, (my thesis advisers)

• Prof.s Brauer, Mackey, Zariski, Tate, Mumford, (I served as a TA in their undergraduate and graduate courses, and learnt a lot from them)
• Prof.s M. Berger (France), and Klingenberg (Germany)
• Prof.s Borel, Atiyah, Milnor, and Chern (IAS, Princeton)
•Prof. Lipman Bers (who brought me down from n-dimensions, to just 2-and 3-dimensions!)
• Prof. Max Zorn (of "Zorn's Lemma", who gave me a view of the French and German mathematical traditions)
• Prof.s Dennis Sullivan, William Thurston, Misha Gromov, and S. -T. Yau (In the US, I shake their hands. In india, I would touch their feet.)
• Prof.s Peter Shalen, Ulrich Pinkall, Hyman Bass, Frank Raymond, K. B. Lee, Ephraim, Constantine, Krishnendu Gongopdhyay, Vikram Aithal, Vikas Jadhav, Rahul Kitture (my collaborators, the last four among my Ph.D. students, from each of whom I have learnt a lot.)

5 A Philosophical Note

Let me come back to the top rectangle in the Tree of Knowledge, starting with "Who Am I?".

You know Descartes' famous statement "I think, therefore I am". This statement occurs in his "Discourse on the Method", the appendix of which contains his fundamental contribution to mathematics, which we call Cartesian Geometry, which says that "Space" and "Number" are two categories of mathematical thought, but they are really two sides of the same coin.

Buddha's first discourse, after he got enlightenment, begins with apparently the opposite: "We are (or we become) what we think".

I think, the word "therefore" in Descartes' statement need not be construed as "implication". I think, Descartes is simply saying that the "existence" is recognized by its thinking capacity. If we go behind the usual dualistic formulation of "recognizing" something by "somebody else", then in my opinion, both Buddha and Descartes are saying the same thing: "Existence" is capable of thinking. So first of all, "Existence", that is "the experience of I", exists! No doubt about it. It comes first, then comes "thinking".

The old Buddhist/Vedantic image of "thoughts" is that they are like waves on the ocean of existence. More precisely, "thoughts" are finite in number, but the mystery of existence is at least as deep as the mystery of the Real Line and Real Numbers. A more precise mathematical image of the relationship of "existence" and "thoughts" is that the existence is like the unit interval $[0, 1]$ of real numbers, and thoughts are like the rational numbers in $[0, 1]$ whose denominators are bounded, say, by $10,000$. The bound depends on the individual. They appear to be filling the whole interval, but they cannot fill the whole interval, for the simple reason that they are finite in number, and the whole interval is not.

Through the process of meditation, you can zoom out the interval $[\frac{1}{10,000}, \frac{1}{9,999}]$, and changing the scale, recognize, "Ah! this was my first thought this morning, then came that thought, ...". So what is the space in between these two rational numbers? It exists. That is all. It is also capable of evolving into more thoughts.

In other words, experience generates thoughts, and thoughts are expressed by a language. The correspondences "experience to thoughts" and "thoughts to expressions" are one-to-many. In particular, the expression is hardly unique. Still, experience is deeper than any of its expressions. In particular, the experience of mathematics is deeper than its set-theoretic expression, although the set-theoretic expression is better than its earlier expressions!

In Buddha's writing, "being" and "becoming" are often not clearly distinguished. So Buddha's "I am what I think" should be really rendered as "I become what I think". So I have made two sentences out of Buddha's one sentence: "I am, so I think", and "I become, what I think." For us, this is quite easy to understand. Dennis thought about Topology, and became a topologist. Hyman thought of Algebra, and became an algebraist. Sagun thought of Analysis, and became an analyst. Shlomo and I thought of Differential Geometry, and we became differential geometers. It is as simple as that.

Coming out of the limited experience-field of Mathematics, we can appreciate Buddha's statement (in Buddha's words):

We become what we think,

With our thoughts we create our worlds,

Think of constructive thoughts, and our life becomes happy,

Think of crooked thoughts, and our life becomes messy,

This is the simple truth of life!

So today, I am very, very happy that all of you have come here. Coming from the Eastern background, I have no problem with the notion of God, as a grand and beautiful hypothesis, which simplifies so much in life. I have seen God in my grand-parents, my parents, especially my mother, my brethren, nephews, nieces, and their children, my wife who was unfortunately plucked away in the blossom of her youth, my teachers, and friends, many of them movers and shakers in the field of mathematics in the last 50 years, which includes all of you.

My salutations to all of you.