The evolutionary robustness of forgiveness and cooperation

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Prisoner's dilemma

A|B	B stays silent (cooperates)	B confesses (defects)
A stays silent (cooperates)	Each serves 1 month	A: 1 year B: goes free
A confesses (defects)	A: goes free B: 1 year	Each serves 3 months

Prisoner's dilemma

Prisoner's dilemma

Payoff Matrix

Prisoner's dilemma

Payoff Matrix Utility function

Prisoner's dilemma

Payoff Matrix Utility function

T>R>P>S

Prisoner's dilemma

Payoff Matrix Utility function

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A|B	COOPERATE	DEFECT
COOPERATE	R, R	S,T
DEFECT	T,S	P,P

Prisoner's dilemma

Payoff Matrix Utility function

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Temptation, Reward, Punishment, Suck

Nash's equilibrium

Nash's equilibrium

Strategies

Nash's equilibrium

Strategies either Defect or Cooperate

Nash's equilibrium

Strategies either Defect or Cooperate

Best response

Nash's equilibrium

Strategies either Defect or Cooperate

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In prisoner's dilemma, the equilibrium is

$$(D, D) = (Defect, Defect)$$

How can cooperation emerge?

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repeation could be an incentive for cooperation
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Player play infinitely many times with fixed probability, δ of playing again.

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$$h_k = ((a^0, b^0), \ldots, (a^j, b^j) \ldots, (a^k, b^k)),$$

$$\hat{h}_k = ((b^0, a^0), \ldots, (b^j, a^j) \ldots, (b^k, a^k))$$

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$$u(a^k, b^k) = T, R, P, S$$

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for δ large, set of possible payoffs is a continuum.

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Grim (cooperate until somebody defects), is an equilibrium

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$$U(g,g) - U(s,g) \geq \delta^k[(1-\delta)[R-T] + \delta(R-P)]$$

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$$U(g,g) - U(s,g) \geq \delta^k[(1-\delta)[R-T] + \delta(R-P)] > 0, \;\; \delta \text{ large}$$

Infinitely many equilibria

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Others “cooperative” equilibria

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Variations of Grim

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For any payoff above (P, P) there is an equilibria

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Continuum of equilibriums

Which one to choose?

Which one to choose?

How to compare those equilibrias?

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Do there exist a selection mechanisms?

Which one to choose?

How to compare those equilibrias?

Do there exist a selection mechanisms?

which is the “optimal“ strategy?

Tournaments

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Axelrod proposed to make tournaments

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Make rankings base in the tournament

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Tournament in 1980,

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Not very nice

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Axelrod: The Evolution of Cooperation

Dynamic enters in town

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Introduce dynamics on the space of strategies

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Which is the appropriate dynamics?

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Replicator dynamic

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Replicator dynamic

$$\mathcal{S} = \{s_1 \dots s_n\}$$

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$$\Delta = \{(x_1 \dots x_n) \in \mathbb{R}^n : x_1 + \cdots + x_n = 1,\ x_j \geq 0, \forall j\}.$$

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$x_i =$ percentage of the population that use the strategy s_i

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Payoff matrix ($\mathcal{S} = \{s_1 .... s_n\}$)

$$A = \begin{pmatrix} a_{11} & \dots & a_{1n} \\ \dots & a_{ii} & \dots \\ a_{n1} & \dots & a_{nn} \end{pmatrix}, \quad a_{ij} = U(s_i, s_j)$$

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$$\dot{x}_j = x_j(\text{payoff of } (s_j) - \text{Average payoff})$$

General version of replicator dynamics

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Large class of dynamics

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$$\dot{x}_j = x_j F[(AX)_j - x^t Ax]$$

General version of replicator dynamics

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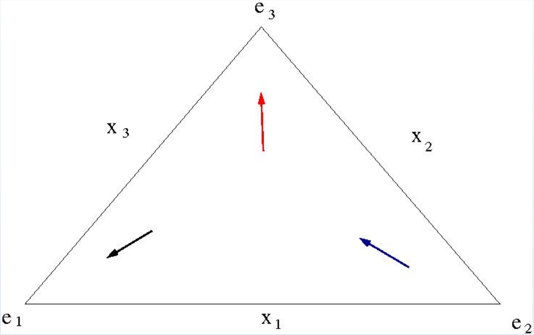
$F : \mathbb{R} \to \mathbb{R}$, strictly increasing function $F(0) = 0$.

Replicator dynamic



Attractors?

Attractors?

Strict Nash equilibria are attractors

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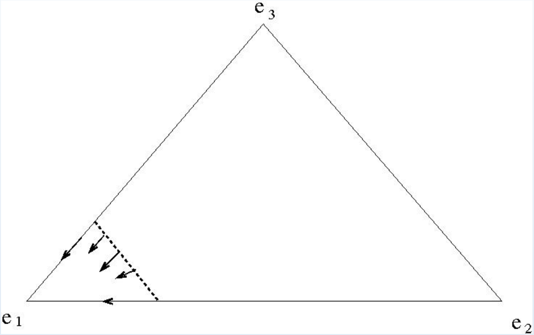
If $U(s, s) > U(s^*, s)$ for any s^* then in any finite population

***s* is an attractor.**

Replicator dynamic



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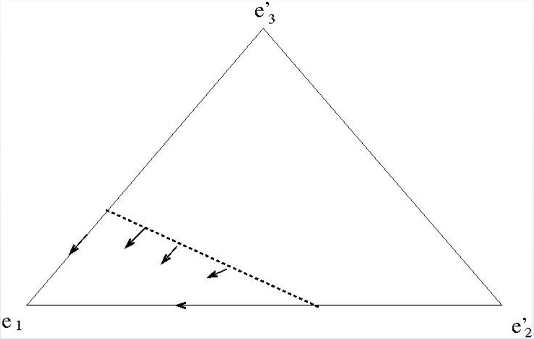
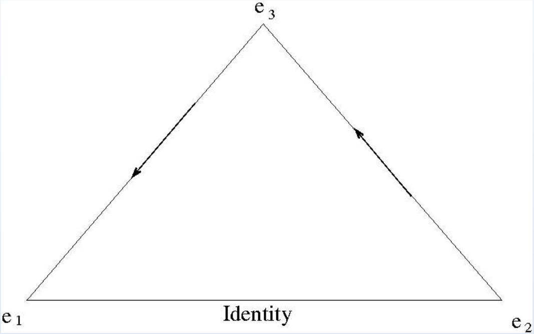


Ties

It does not exist s such that $U(s,s) > U(s^*,s)$ for any s^*

Fooling/upseting strategies (Grim and always cooperate)



Infinitely Repeated Prisoner's dilemma with trembles

TREMBLES:Probability of small mistakes $1-p, p$

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All possible histories are considered. A tree

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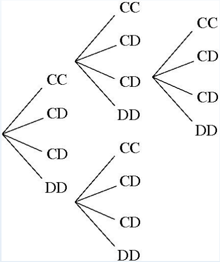

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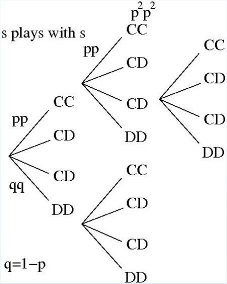

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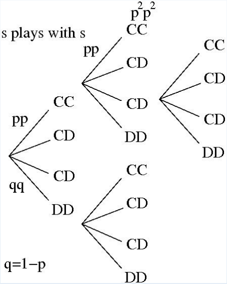

$$U(s_1, s_2) = (1 - p^2\delta) \sum_{h_k} P_{s_1,s_2}(h_k)\delta^k u(a^k, b^k).$$

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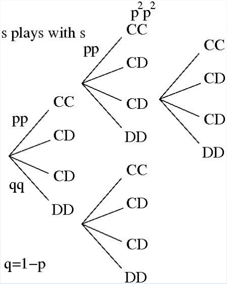

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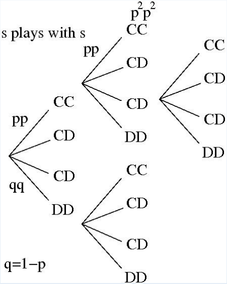

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All the paths are explore with some probability
Intended paths are more probable.

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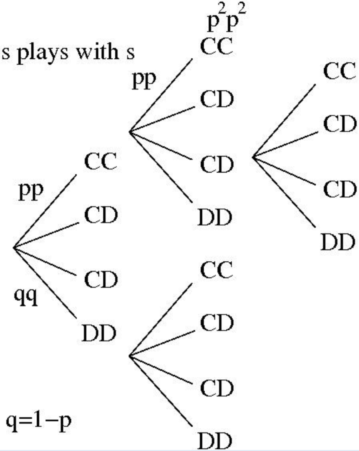

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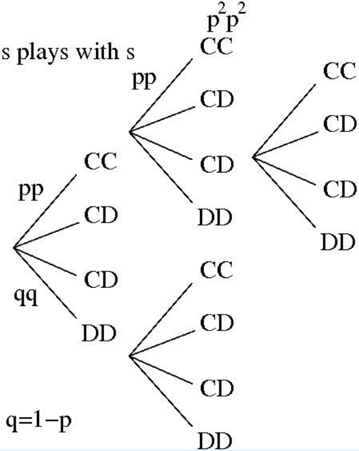

$$U(s, s) - U(s^*, s)$$

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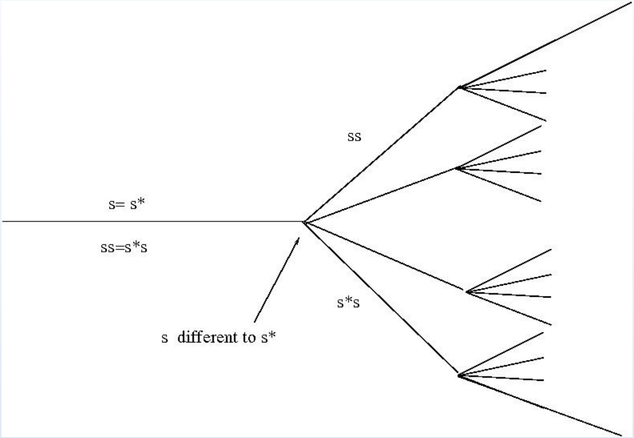

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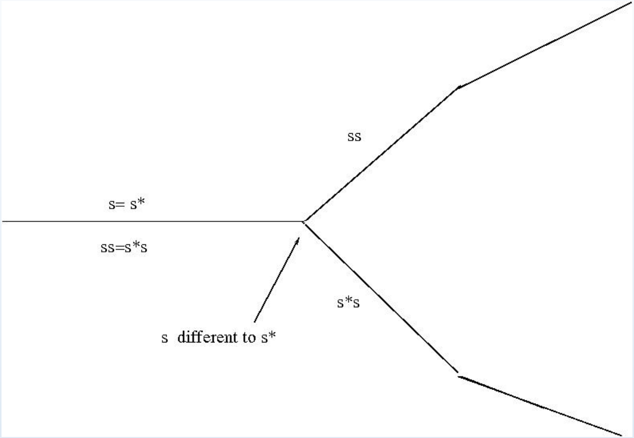

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TREMBLES BREAK TIES.

Still, there are infinitely many of Strict Nash equilibria

Which are the attractors frequently choosen

Which are the attractors frequently choosen

Which attractors are frequently choosen in any finite population?

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Does there exist a strategy, such that in any population

if a fixed fraction use that strategy

then that strategy is going to become dominant?

Which are the attractors frequently choosen

Which attractors are frequently choosen in any finite population?

Does there exist a strategy, such that in any population

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To have a large basin of attraction in a uniform way
regardless of the population

Dynamic re-enters: Uniform Large Basin of Attraction

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$$B_{K_0}(s) \subset B^s_{loc}(s).$$

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$$B_{K_0}(s) \subset B^s_{loc}(s).$$

Does it exist such strategy?

Dynamic re-enters: Uniform Large Basin of Attraction

s has **ULBA,**
if there exists $K_0 := K_0(s)$ such that

for any δ large and p large,

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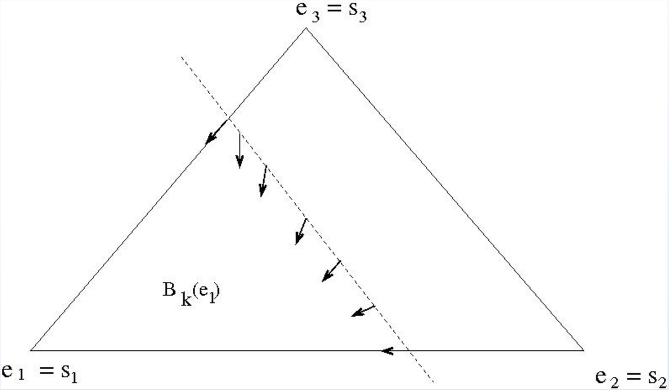
Does it exist such strategy?

Which properties should satisfy?

Uniform Large Basin of Attraction



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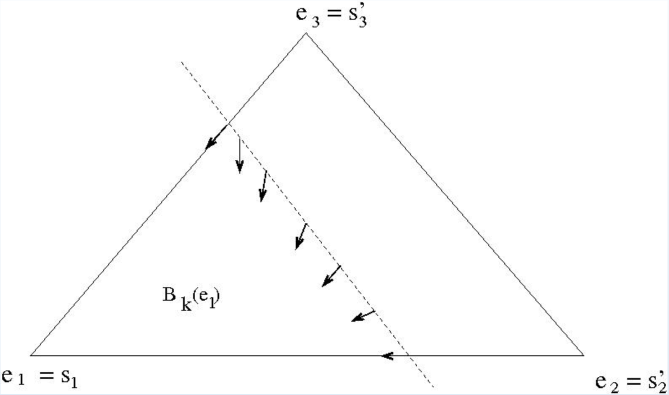

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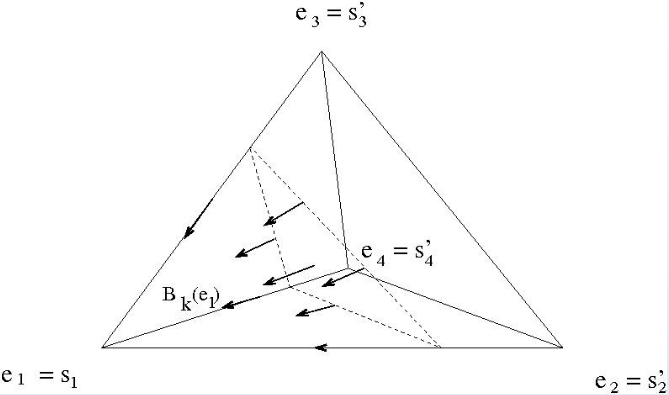

Super-game

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provided that they are kept in the changes of populations.

Conditions to have ULBA

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Given $\{s, s^*, s'\}$

$$\frac{U(s^*, s') - U(s, s') + U(s', s^*) - U(s, s^*)}{U(s, s) - U(s^*, s)} < M_0$$

the ball of size $\frac{1}{M_0}$ is in the basin of s (against s', s^*)

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If the supreme over al strategies is smaller than $M_0 < \infty$

$B_{\frac{1}{M_0}}(s)$ is in the basin of attraction of s in any finite population

Conditions to have ULBA

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If s has a uniform large basin in any set of THREE strategies

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It is not enough to do well againts attack by individual strategies

Proof: Back to replicator equation

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size of the basin on the edges $\frac{1}{1+\frac{a_{ii}-a_{ji}}{a_{jj}-a_{ij}}}$.

Back to replicator equation

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Given a positive $K_0 < 1$,

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"Properties" on any matrix $A \in \mathbb{R}^{n \times n}$ (arbitrary n) s.t. for the equation

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"Properties" on any matrix $A \in \mathbb{R}^{n\times n}$ (arbitrary n) s.t. for the equation

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the vertex e_1 has a basin of attraction containing the ball of radius K_0.

$$B_{K_0}(e_1) \subset B^s_{loc}(e_1).$$

Back to replicator equation

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$$M_0 = \max_{i,j \geq i} \{ \frac{M_{ij} + M_{ji}}{-N_i}, \ 0 \}.$$

Then,

$$B_{\frac{1}{M_0}}(e_1) = \{ \bar{x} : \sum_{i \geq 2} x_i \leq \frac{1}{M_0} \} \subset B^s_{loc}(e_1).$$

Replicator equation/adaptation to games

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Given $\mathcal{S} = \{s_1 \dots s_n\}$, let M be the payoff matrix

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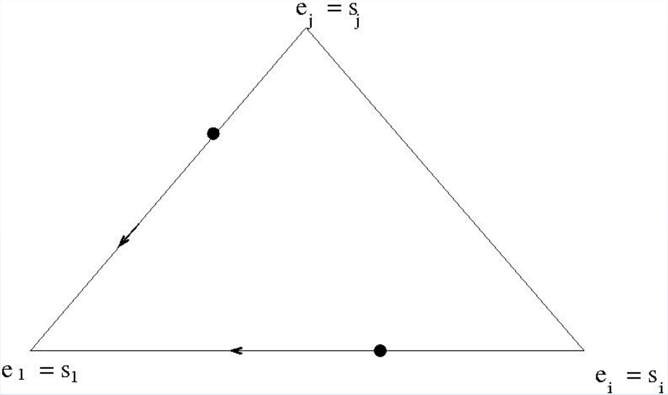
Replicator equation



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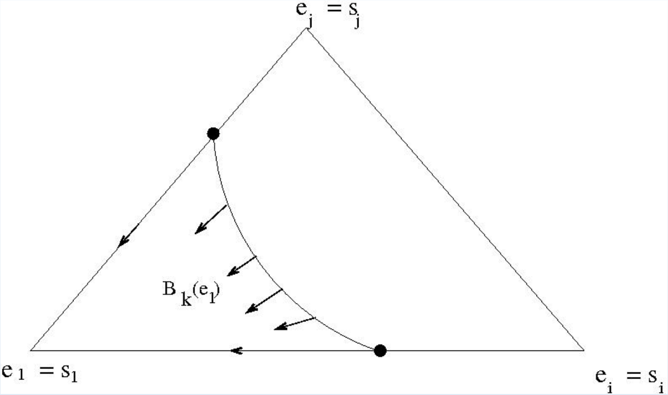

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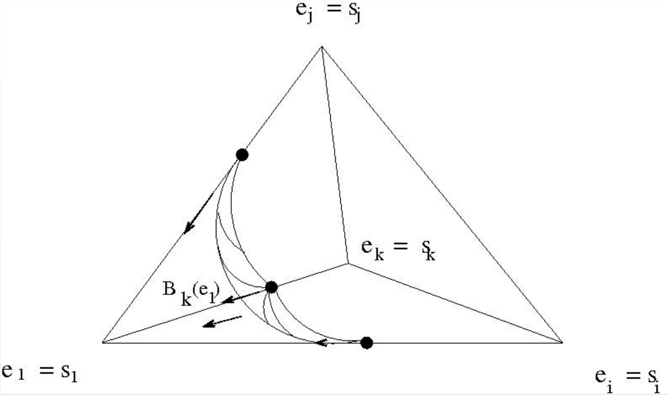

How to compute?

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Cross ratio.

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Given a strategy s and another one s^*

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Cross ratio condition and $T + P < 2R \to$ ULBA

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Meaning?

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unforgiving strategies lose in payoffs relative to strategies that forgive

Star-type strategies

Star-type strategies

given s^*, s and a seed history h_k, we look to the equilibrium path $h_{s^*,s/h_k}$

$$(a^1, b^1), (a^2, b^2), \dots (a^t, b^t) \dots$$

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frequencies of s^* getting R, S, T, P whenever plays with s with seed h_k

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Star-type strategies

given s^*, s and a seed history h_k it is generated history

b_1 frequency after h_k of playing (C, C) (s^* gets R)

b_2 frequency after h_k of playing (C, D) (s^* gets S)

b_3 frequency after h_k of playing (D, C) (s^* gets T)

b_4 frequency after h_k of playing (D, D) (s^* gets P)

Star-type strategies

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Condition 1: s is a **Cooperative strategy** (efficient)

After any path, (s, s) eventually goes back to a path of cooperation

Trying to move away from low score

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If s^* defected when s cooperated then s has to retaliate.

Examples of Star strategies

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RETALIATION s retaliate to a defection of s^* while s cooperated

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What about PAYBACK?

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COOPERATION

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What about PAYBACK?

Offer cooperation after taking advantage

Win-Stay-Lose-Shift

Win-Stay-Lose-Shift

if $C, C \longrightarrow C$,
COOPERATE

Win-Stay-Lose-Shift

if $C, C \longrightarrow C$,
COOPERATE

if $D, D \longrightarrow C$,
TRY TO MOVE AWAY FROM MUTUAL DEFECTION, FORGIVE

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if $C, D \longrightarrow \ D$,

PUNISH

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Generalized forms/ CLASS OF WSLS

Win-Stay-Lose-Shift

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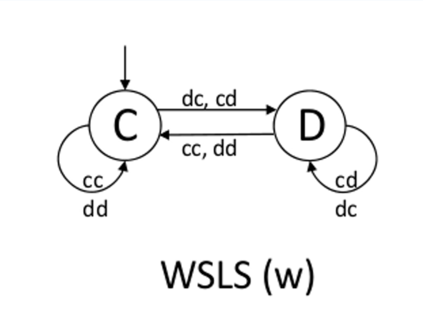
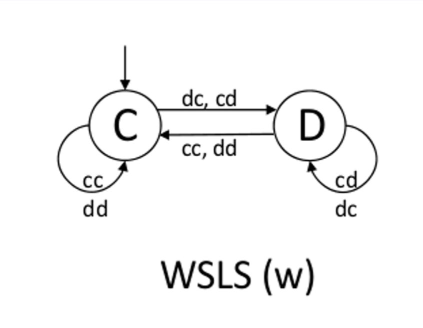
Generalized forms/ CLASS OF WSLS

WSLS is a START-TYPE strategy

Win-Stay-Lose-Shift

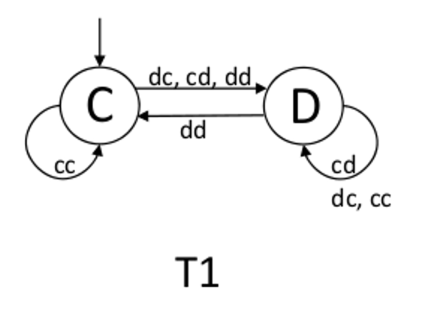


Win-Stay-Lose-Shift



WSLS is a START-TYPE strategy

Trigger strategies



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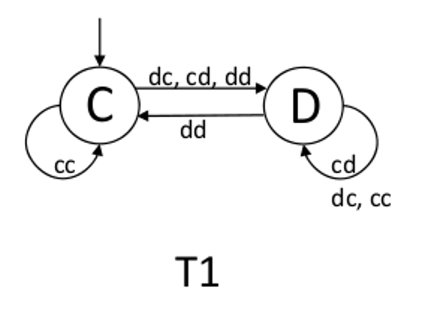

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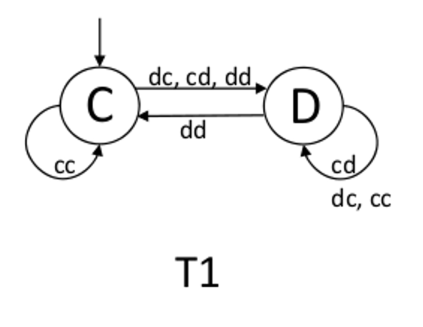
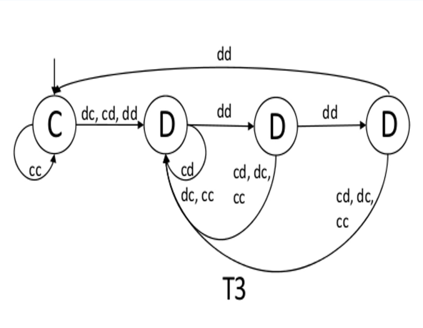


Trigger is a START-TYPE strategy

Carefull WSLS

Trigger strategies



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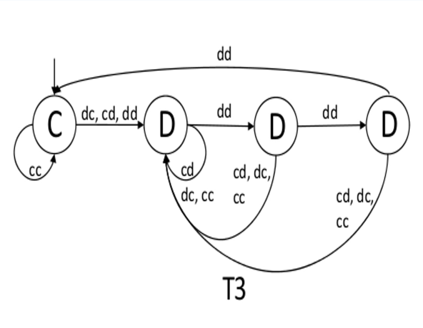
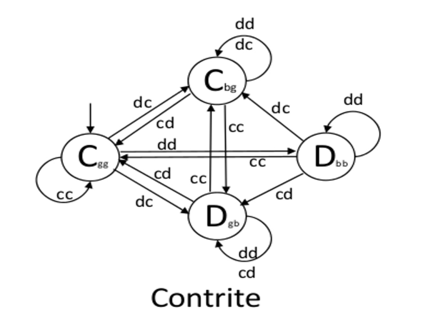
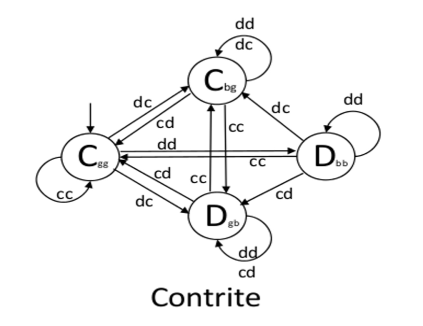


Trigger-n is a START-TYPE strategy

Contrite strategies



Contrite strategies



Contrite is a START-TYPE strategy

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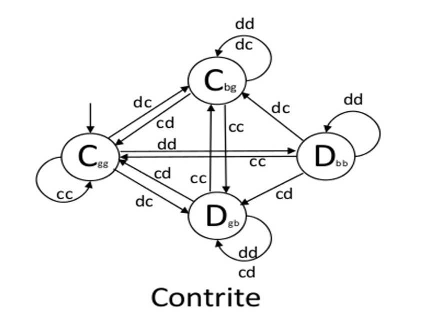

Contrite is a START-TYPE strategy

Contrite PAYS BACK

Similarities and Differences

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COOPERATE

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Try to move away from low score

Intend to go back to cooperation

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WSLS take advantages

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CONTRITE pays back

Relations to experiment

Those type of strategies does not appear in the experiments

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Can we test that?

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How to design an experiment to test the theoretical's predictions

Characterizing strategies that has ULBA

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We show sufficient conditions for a strategy to be sucessful.

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Axelrod's conjecture:

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- Retaliating;
- Forgiving;
- Non-envious.

ULBA implies retaliation

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If s is Nash equilibrium, it has to punish a defection

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Always Cooperate is not an equilibrium

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ULBA implies Cooperate (with itself after any path)

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If s is ULBA, then for any history h_t follows that

$$\lim_{\delta \to 1, p \to 0} U(s, s/h_t) = R.$$

Questions?

Experiments?

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- topological and differentiable structure on set of strategies;

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Mutations? Dynamics accepting mutations

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Caviat: Mutations of equilibria is not necessary an equilibria

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