Conformal actions of hyperbolic lattices and 4-cobordisms

Boris N. Apanasov (Univ. of Oklahoma, apanasovou.edu)

We discuss how the global geometry and topology of manifolds depend on different conformal group actions of their fundamental groups, and in particular, how the properties of a non-trivial compact 4-dimensional cobordism M whose interior has a complete hyperbolic structure depend on properties of the variety of discrete representations of the fundamental group of its 3-dimensional boundary ∂M. In addition to the standard conformal ergodic action of a uniform hyperbolic lattice on the round sphere S^{n-1} and its quasiconformal deformations in S^n, we present several constructions of unusual actions of such lattices on everywhere wild spheres (boundaries of quasisymmetric embeddings of the closed n-ball into S^n), on non-trivial $(n-1)$-knots in S^{n+1}, as well as actions defining non-trivial compact cobordisms with complete hyperbolic structures in its interiors. Such unusual actions correspond to discrete representations of a given hyperbolic lattice from "non-standard" components of its varieties of representations (faithful or with large kernels of defining homomorphisms).

In particular we construct such non-trivial hyperbolic 4-cobordisms M whose boundary components have a great geometric symmetry. These 3-dimensional boundary manifolds are covered by the discontinuity set $\Omega(G) \subset S^3$ with two connected components Ω_1 and Ω_2, where the action Γ of the fundamental group $\pi_1(\partial M)$ is symmetric and has contractible fundamental polyhedra of the same combinatorial (3-hyperbolic) type. Nevertheless we show that a geometric symmetry of boundary components of our hyperbolic 4-cobordism $M(G))$ are not enough to ensure that the group $G = \pi_1(M)$ is quasi-Fuchsian, and in fact our 4-cobordism M is non-trivial. This is related to non-connectedness of the variety of discrete representation of the uniform hyperbolic lattice Γ and infinite kernels of the constructed homomorphisms $\Gamma \to G$, and also gives a new view on Andreev's hyperbolic polyhedron theorem.

Curve intersections and hyperbolic structures

Ara Basmajian (Hunter College and CUNY Graduate Center)

Let m be a hyperbolic structure on a surface S and γ a free homotopy class of a closed curve on S; denote the m-length of the closed geodesic in the homotopy class of γ by $L_\gamma(m)$. In this talk we will discuss our ongoing study of the relationship between $L_\gamma(m)$ and the combinatorial/topological type of γ in various contexts. In particular, our investigation of $L_\gamma(m)$ when γ varies over all combinatorial types with a fixed self-intersection number and,

1. m is a fixed hyperbolic structure.
2. m varies in the Teichmüller space.
3. m varies through all hyperbolic structures on all topological surfaces.

Of course self-intersection number is one invariant of combinatorial type. We will finish with a discussion of others and their relationship to the length function.

A differential operator approach to the Jacobian Conjecture

Hyman Bass (University of Michigan)

The Jacobian Conjecture (Keller, 1939) says that a polynomial map $F : C^n \to C^n$ of Jacobian determinant 1 is bijective. This problem has been approached by a variety of methods, yielding many partial results, but it remains open for all $n \geq 2$. I will describe an approach using algebraic differential operators that, in the case of dimension 2, makes some novel use of Diophantine geometry and classical analysis.

Closed Aspherical Manifolds with Center: A counterexample to the Conner-Raymond Conjecture

Sylvain Cappell (Courant Institute, NYU)

Borel and Conner-Raymond showed a half-century ago that an aspherical manifold with a non-trivial circle action must have non-trivial center in its fundamental group, i.e., given by an orbit circle. Conner and Raymond had then conjectured that, conversely, an aspherical manifold with non-trivial center would always have a non-trivial circle action. Using a synthesis of methods of geometric group theory, constructions from hyperbolic geometry and equivariant surgery theory, we construct counterexamples, i.e., closed aspherical manifolds with center but no topological group action. This is joint work with Shmuel Weinberger (U. of Chicago) and Min Yan (Hong Kong U. of Sci. & Tech.).

Embedding CR Three Manifolds, Sharp Eigenvalue Estimates for the Kohn Laplacian and CR Yamabe Invariants.

Sagun Chanillo (Rutgers University, New Brunswick)

We consider a compact, three dimensional CR (Cauchy-Riemann)manifold with no boundary. A CR manifold is a Contact manifold with an almost complex structure on the Contact planes. In the strongly pseudoconvex case one can define a natural Laplacian, the Kohn Laplacian. One can also define connections and curvature for the metric arising from the Levi form. This is the Webster connection and curvature. There is also a conformally covariant operator, the Paneitz operator attached to such manifolds. We obtain a sharp lower bound for the first eigenvalue of the Kohn Laplacian and relate this to embedding abstract CR manifolds in Complex space. The eigenvalue bounds are the exact analogs of Lichnerowicz's well-known bounds from the 1950's for the first eigenvalue of Laplace's operator on compact Riemannian manifolds. This is joint work with Hung-Lin Chiu and Paul Yang.

Computer driven theorems and questions in geometry

Moira Chas (SUNY Stony Brook)

Given an orientable surface S with negative Euler characteristic, a minimal set of generators of the fundamental group of S, and a hyperbolic metric on S, each free homotopy class C of closed oriented curves on S, determines three numbers: the word length (that is, the minimal number of generators and inverses necessary to express C as a cyclically reduced word), the minimal self-intersection and the geometric length.

One the other hand, the set of free homotopy classes of closed directed curves on S (as a set) is the basis of a Lie algebra structure (discovered by Goldman). This Lie algebra is closely related to the intersection structure of curves on S.

These three numbers, as well was the Goldman Lie bracket of two classes, can be explicitly computed (or approximated) by the help of computer. These computations lead us find counterexamples to existing conjectures and to establish new conjectures.

For instance, we conjectured that the distribution of self-intersection of classes of closed directed curves on a surface with boundary, sampling by word length, appropriately normalized, tends to a Gaussian when the word length goes to infinity. Later on, jointly with Lalley, we proved this result. Recently, Wroten extended this result to closed surfaces.

In another direction, the computer allowed to us to study the relation between self-intersection of curves and length-equivalence. (Two classes a and b of curves are length equivalent if for every hyperbolic metric m on S, m(a)=m(b).)

Finally, the computer allowed us to probe conjectures about how the intersection structure of curves on S is "encoded" in the Goldman Lie algebra.

Difference equations on infinite graphs

Jozef Dodziuk (Queens College and CUNY Graduate Center)

I will review several examples where techniques motivated by ideas in Partial Differential Equations and Differential Geometry lead to interesting results about difference equations on graphs, most notably the combinatorial Laplacian. Examples will include the maximum principle, Harnack inequality, Cheeger's inequality and a recent result about surjectivity of combinatorial Laplacian for infinite graphs.

Plumbing paired punctures in Kleinian groups

Clifford Earle (Cornell University)

Many years ago, Al Marden and I applied a plumbing technique to certain Kleinian groups, thus putting so-called Earle-Marden coordinates on compactified moduli space.

This talk, continuing our joint work, will describe how our plumbing technique can be applied to a wide class of "Kleinian plumbing groups". The simplest such group is a well-known subgroup of the classical modular group. We shall describe in detail the result of plumbing this group.

Extremal embedded cylinders

FREDERICK P. GARDINER (CUNY Graduate Center)

Extremal cylindrical differentials play a central role for understanding any Riemann surface X. For a conformal embedding of a cylinder C into X with height b and circumference a the ratio $M(C) = a/b$ is a conformal invariant of the particular cylinder C. We say C is *extremal* in its homotopy class Γ if the ratio a/b is as small possible among all homotopically equivalent embeddings, and put

$$M(\Gamma) = \inf_{C \in \Gamma} \{M(C)\}.$$

By taking this infimum, the number $M(\Gamma)$ becomes a conformal invariant of X and the homotopy class of Γ. $M(\Gamma)$ is realized either by holding a fixed and making b as large as possible or by holding b fixed and making a as small as possible.

For any non-trivial homotopy class you can always find such an extremal cylinder. Here are three amazing facts:

1. This cylinder is uniquely determined by the Riemann surface X and the homotopy class of Γ.

2. It corresponds to a unique quadratic differential $q(z)(dz)^2$ holomorphic on X whose restriction to the maximal cylinder C is equal to

$$-\left(\frac{1}{2\pi}\right)^2 \left(\frac{dz}{z}\right)^2,$$

   where z is a conformal map from the maximal cylinder C to the region between two concentric circles in the complex plane.

3. $\log M(\Gamma)$ is a differentiable function on the Teichmüller space of X and its derivative is expressed by the quadratic differential q.

Parsing out the possilities that occur when $M(\Gamma) > 0$ and $M(\Gamma) = 0$, you can arrive at and develop the following topics in Riemann surface theory:

- The Riemann mapping theorem.

  (X equals a simply connected surface, $Y = X - \{a\ disc\}$, $M(\Gamma, Y) > 0$, Γ equals the family of closed curves in Y homotopic to the boundary of the disc).

- The generalized Riemann mapping theorem and Koebe's uniformization theorem.

  ($M(\Gamma) \geq 0$).

- Slodkowski's extension theorem for holomorphic motions.

  ($M(\Gamma) = 0$ applied to $X - \{p_0\}$ where Γ is the family of curves in X surrounding p_0).

- The Weirstrass P-function.

  (X equals a torus, p a point on the torus, Γ is the family of curves in X surrounding p, the maximal cylinder C in X lifts to a hexagonal fundamental domain, and $P(z)(dz)^2$ is the holomorphic quadratic differential).

- The extremal length embedding of Teichmüller space in the space of projective classes of extremal lengths of homotopy classes Γ.

  (X equals a surface, X_τ is X marked surface with the complex structure τ and the embedding is

  $$Teich(X) \ni \tau \mapsto (M(\Gamma, X_\tau))_{\Gamma \in G} \in \mathbb{R}_+^G / \mathbb{R}_+,$$

  where Γ varies over the family of homotopy classes G of simple closed curves on X).

- The minimum Dirichlet principle for measured foliations.

  (F is any measured foliation in the topological closure of G with the weak topology induced by intersection numbers and q is the measured foliation with minimum Dirichlet integral whose heights coincide with the heights of F).

- The minimal axis theorem.

  (F_1 and F_2 are two measured foliations on a surface of finite analytic type that are weakly transverse in the sense that there is a number $\epsilon_0 > 0$ such that for every essential Γ either the height of Γ over F_1 or over F_2 is greater than or equal to ϵ_0. Then q is a holomorphic differential that determines a unique Teichmüller axis where the product $M(X_\tau, F_1)M(X_\tau F_2)$ is minimum).

LARGE AUTOMORPHISM GROUPS

JANE GILMAN, RUTGERS-NEWARK

Let S be a compact Riemann Surface of genus $g \geq 2$ and let G be a group of conformal automorphisms of S of order n. Kulkarni introduced the notion of a *large automorphism group*. The group G is large if $n > 4(g-1)$. If G is large, then the quotient surface $S_0 = S/G$ is either a surface of genus 0 with appropriate branching or a surface of genus 1 which is branched over three points. We discuss various results about the action of a large automorphism group on the fundamental group of S and the first homology group. We find applications to the representation variety.

Three-dimensional complete affine manifolds

William Goldman (University of Maryland)

In this talk I will describe recent advances in the classification and geometric understanding of quotients of Euclidean 3-space by discrete groups of affine transformations, and their relationship to hyperbolic geometry in dimension 2 and Lorentzian geometry in dimension 3.

Local Quasiconformal Motions and Universal Property

Yunping Jiang (City University of New York)

In this talk, I will discuss a recent work jointly with Sudeb Mitra, Hiroshige Shiga, and Zhe Wang on the extension problem and the universal property in continuous motions, quasiconformal motions, local quasiconformal motions, and holomorphic motions. In this work, we give an example of a quasiconformal motion of an infinite set in the Riemann sphere, over an interval, that cannot be extended to a quasiconformal motion of the sphere. Following this example, we introduce a new concept called local quasiconformal motion. With this new definition, we show that any local quasiconformal motion of a set, over a simply connected Hausdorff space, can be extended to a quasiconformal motion of the whole sphere, over the same parameter space. Furthermore, we show that this can be done in a conformally natural way. We relate these questions to the universal property of the Teichmuller space of a closed set in the sphere. We also prove that a differentiable quasiconformal motion is a local quasiconformal motion. Finally, we will discuss a conjecture which says that a guiding quasiconformal motion is a local quasiconformal motion.

Thurston Rigidity for some classes of transcendental maps

Linda Keen (Graduate Center and Lehman College CUNY)

Thurston's theorem for rational maps says that a finite degree covering map of the Riemann sphere whose branch points have finite orbits can be "realized" by an essentially unique rational map unless there is a topological obstruction. The obstruction also has analytic and geometric characterizations. In this talk, based on joint work with Tao Chen and Yunping Jiang, we discuss extending Thurston's theorem to infinite degree covering maps with finite post-singular sets. We show that for special families the Thurston characterization carries over to infinite degree maps.

Deformation theory and finite simple quotients of triangle groups

Alex Lubotzky (Hebrew University)

Many works have been dedicated to the question: given a hyperbolic triangle group $T = T(a, b, c)$, which finite (simple) groups are quotients of T? The case of $(a, b, c) = (2, 3, 7)$ being of special interest. Many positive and negative results have been proven by either explicit or random methods. We will present a new method to study this problem using deformation of group representations. This will enable us to 1) prove a conjecture of Marion which gives a guiding rule for the wealth of results 2) adding many new groups to the list of quotients of T. This includes the case of (2.3.7), for which we saw that simple groups of type E_8 are quotients of it (answering a question of Guralnick). This is a joint work with Michael Larsen and Claude Marion (to appear in JEMS).

Discrete Smoke

Ulrich Pinkall (Technische Universitt, Berlin, Germany)

We present an algorithm to approximate a given velocity field in space ("smoke") by the field generated by a finite number of closed vortex filaments (smoke rings). This discretization of smoke into smoke rings is useful both for visualization and for fluid simulation.

A two-dimensional version of this algorithm (which also has applications in Computer Graphics) concerns choosing optimal positions for the branch points of certain branched covers of a given Riemann surface."

Infinitesimal Liouville currents, cross-ratios and intersection numbers

Dragomir Saric (Queens College and CUNY Graduate Center)

Many classical objects on a surface S can be interpreted as cross-ratio functions on the circle at infinity of the universal covering. This includes closed curves considered up to homotopy, metrics of negative curvature considered up to isotopy and, in the case of interest here, tangent vectors to the Teichmüller space of complex structures on S. When two cross-ratio functions are sufficiently regular, they have a geometric intersection number, which generalizes the intersection number of two closed curves. In the case of the cross-ratio functions associated to tangent vectors to the Teichmüller space, we show that two such cross-ratio functions have a well-defined geometric intersection number, and that this intersection number is equal to the Weil-Petersson scalar product of the corresponding vectors. This is a joint work with Francis Bonahon.

On some analytic properties of deformation spaces of Kleinian groups

Hiroshige Shiga (Tokyo Institute of Technology, Japan)

Let G be a non-elementary Kleinian group. We consider the space of quasi-conformal deformations of G. The space has a natural complex structure and it is finite dimensional if G is finitely generated. In this talk, we consider complex analytic properties of the spaces, which are related to some results by Bers, Kra-Maskit and McMullen.

Kulkarni's conformal boundary for simply connected Lorentz surfaces

Robert W. Smyth (Cooper Union, NY)

This talk will review Kulkarni's conformal boundary for simply connected Lorentz surfaces, its topology and its use in discriminating conformally inequivalent Lorentz surfaces. A completion of the conformal boundary using a construction analogous to Caratheodory's prime ends will also be introduced.

Recent incarnations of the Cartan three-form

Shlomo Sternberg (Harvard University)

I will recall a few results from my paper with Kostant: Symplectic Reduction, BRS Cohomology, and Infinite-Dimensional Clifford Algebras *ANNALS OF PHYSICS, 176, 49-113 (1987)* and discuss some recent developments due to Li-Bland and Meinrenken extending these results to the construction and properties of Courant algebroids

From Topology to Anslysis

Dennis Sullivan (CUNY Graduate Center and SUNY Stony Brook)

Consider dividing space into cubes and subdividing again and again. The cells of each subdivision after the first have a natural partial semi group structure. This gives an associative algebra structure on the real cochains at each level for which the coboundary operator is a derivation. Natural transversal inequalities imply these various products have a limit under subdivision. The limit is graded commutative and is consistent with the wedge product of differential forms.

Each subdivision has a poincare dual cell operator and adding in dual tangential inequalities imply these converge to an operator consistent with the hodge star.

One may now write a system of finite dimensional ODEs associated to infinite dimensional PDEs. One may use this on two ways: First, one may try to study the PDE for solvability or blow up using these nonlinear ODE approximations and the finite dimensional theory of dynamics. Second, one may use these ODEs as effective models of the physical process the PDE is meant to model exactly. This is reasonable even when the PDE itself is divergent or intractable.The motivating example is the Navier Stokes model of incompressible fluid motion.