1 0-cell
1 1-cell
2 2-cells

$\chi(S^2) = 1 - 1 + 2$
$= 2$

$H_0(S^2) = \mathbb{Z}$ dim = 1
$H_1(S^2) = \{0\}$ dim = 0
$H_2(S^2) = \mathbb{Z}$ dim = 1

1-handles

0-handle

$\chi(T^2) = 1 - 2 + 1$
$= 0$

$H_0(T^2) = \mathbb{Z}$

$H_1(T^2) = \mathbb{Z} \oplus \mathbb{Z}$

$H_2(T^2) = \mathbb{Z}$

$P^2 =$  $\underset{S^1}{\cup}$ 

$\chi(P^2) = 1 - 1 + 1$
$= 1$

$H_0(P^2;\mathbb{Z}) = \mathbb{Z}$
$H_1(P^2;\mathbb{Z}) = \mathbb{Z}/2$
$H_2(P^2;\mathbb{Z}) = \{0\}$

?

But with field coefficients

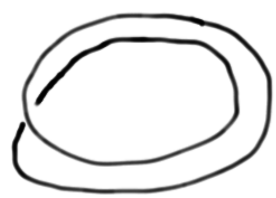
$H_0(P^2;\mathbb{Q}) = \mathbb{Q}$		$H_0(P^2;\mathbb{Z}/2) = \mathbb{Z}/2$
$H_1(P^2;\mathbb{Q}) = \{0\}$	OR	$H_1(P^2;\mathbb{Z}/2) = \mathbb{Z}/2$
$H_2(P^2;\mathbb{Q}) = \{0\}$		$H_2(P^2;\mathbb{Z}/2) = \mathbb{Z}/2$

χ is additive:

$$\boxed{\chi(X \cup Y) = \chi(X) + \chi(Y) - \chi(X \cap Y)}$$

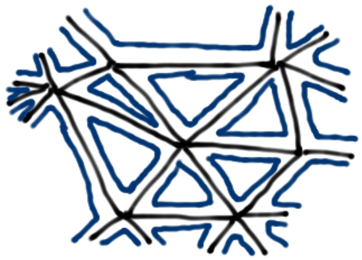
Now $\chi(S^1) = 0$, $\chi(D^2) = 1$. So:

$$\boxed{\chi(F \# G) = \chi(F) + \chi(G) - 2}$$



$\underline{\mathrm{Th}^m}$ Every closed surface can be triangulated.

Triangulations $\rightsquigarrow$ handle decompositions



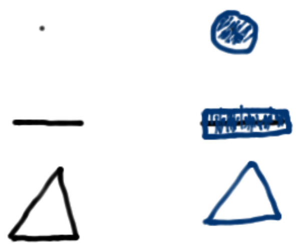
Poincaré duality

Cell decomposition

vertices $\longleftrightarrow$ 2-cells

edges $\longleftrightarrow$ edges

cocycles $\longleftrightarrow$ cycles



0 cells $\rightsquigarrow$ 0 hande $D^0 \times D^2$

1 cels $\rightsquigarrow$ 1 houle $D^1 \times D^1$

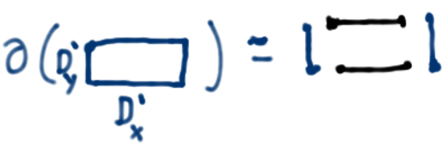
2 cells $\rightsquigarrow$ 2 handle $D^2 \times D^0$

n-dim'l k-handle : $D^k \times D^{n-k} \cong D^n$

attached along $\partial(D^{n-k}) \times D^k$

$$\partial(D^k \times D^l) = \partial D^k \times D^l \bigcup_{\partial D^k \times \partial D^l} D^k \times \partial D^l$$

$\partial(D^1 \times D^1) =$



Th^m Every closed surface is homeomorphic (PL, smooth) to connected sum of tori or projective planes. (Homeo classes of) Surfaces form a commutative monoid under connected sum, with identity S^2, generators T, P and a single relation: $P \# P \# P = T \# P$.

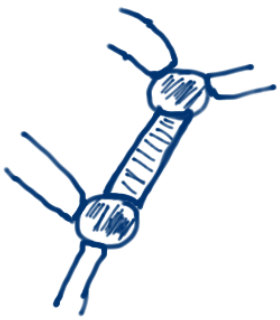
$S \backslash \chi$	2	1	0	-1	-2	-3	-4
Orientable	S^2		T		$2T$		$3T$
Non orientable		P	$2P$	$3P$	$4P$	$5P$	$6P$

$$\chi(F \# T) = \chi(F) - 2 \Rightarrow \chi(\underbrace{T \# \cdots \# T}_{g}) = 2 - 2g$$

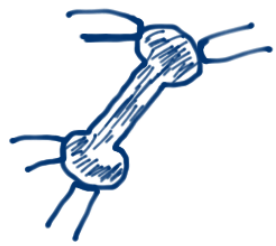
$$\chi(F \# P) = \chi(F) - 1 \Rightarrow \chi(\underbrace{P \# \cdots \# P}_{p}) = 2 - p$$

1) Every closed surface has a handle decomposition with <u>one</u> 0-handle and <u>one</u> 2-handle

Take a spannig tree of the 1-skeleton and combine 0-handes and 1-handles allong this spanning tree to form a single 1-handle.



two 0-handles
and one 1-handle



one 0-handle

Dually we can combine 2-handles and 1-handes along a spanning tree of the dual cell decomposition to get a single 2-handle.

two 2-handles
one 1-handle

one 2-handle

2) So look at how we can attach 1-handles to a 0-handle

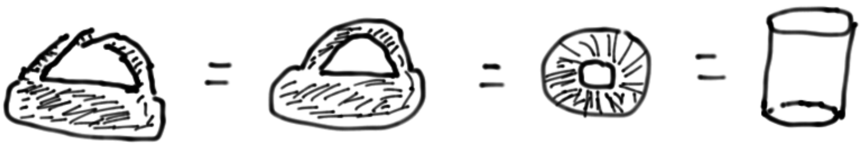
Two kinds of 1-handle attaching:

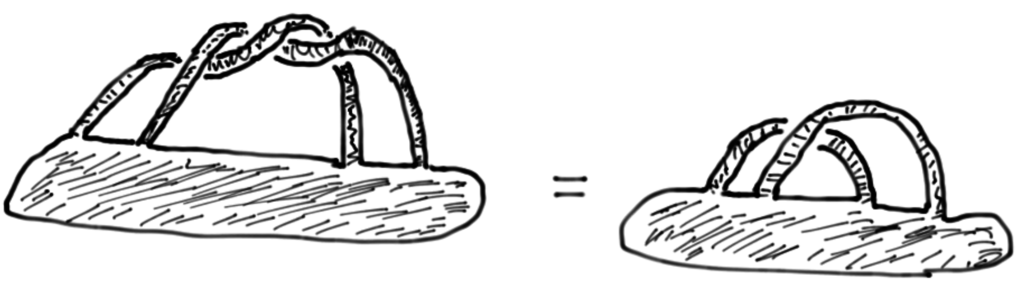

Straight

vs


twisted

Up to homeo:





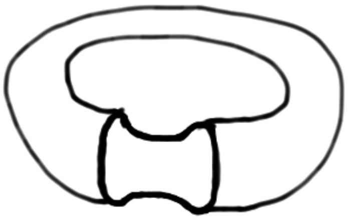
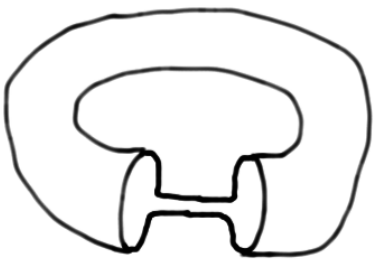
3) Handle sliding etc.

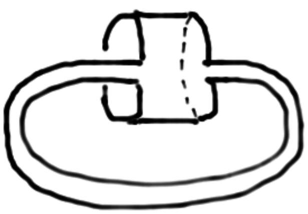
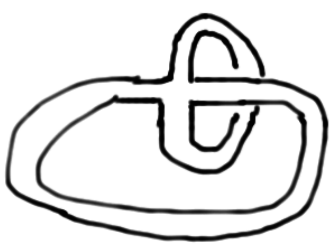
Since we have a single 2-handle straight 1-handles come in interlocking pairs.

Two ∂ comp.

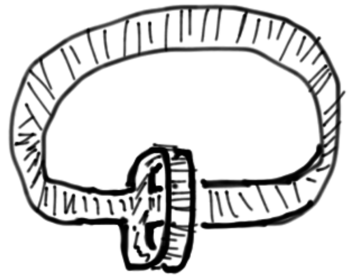
Three ∂ comp.

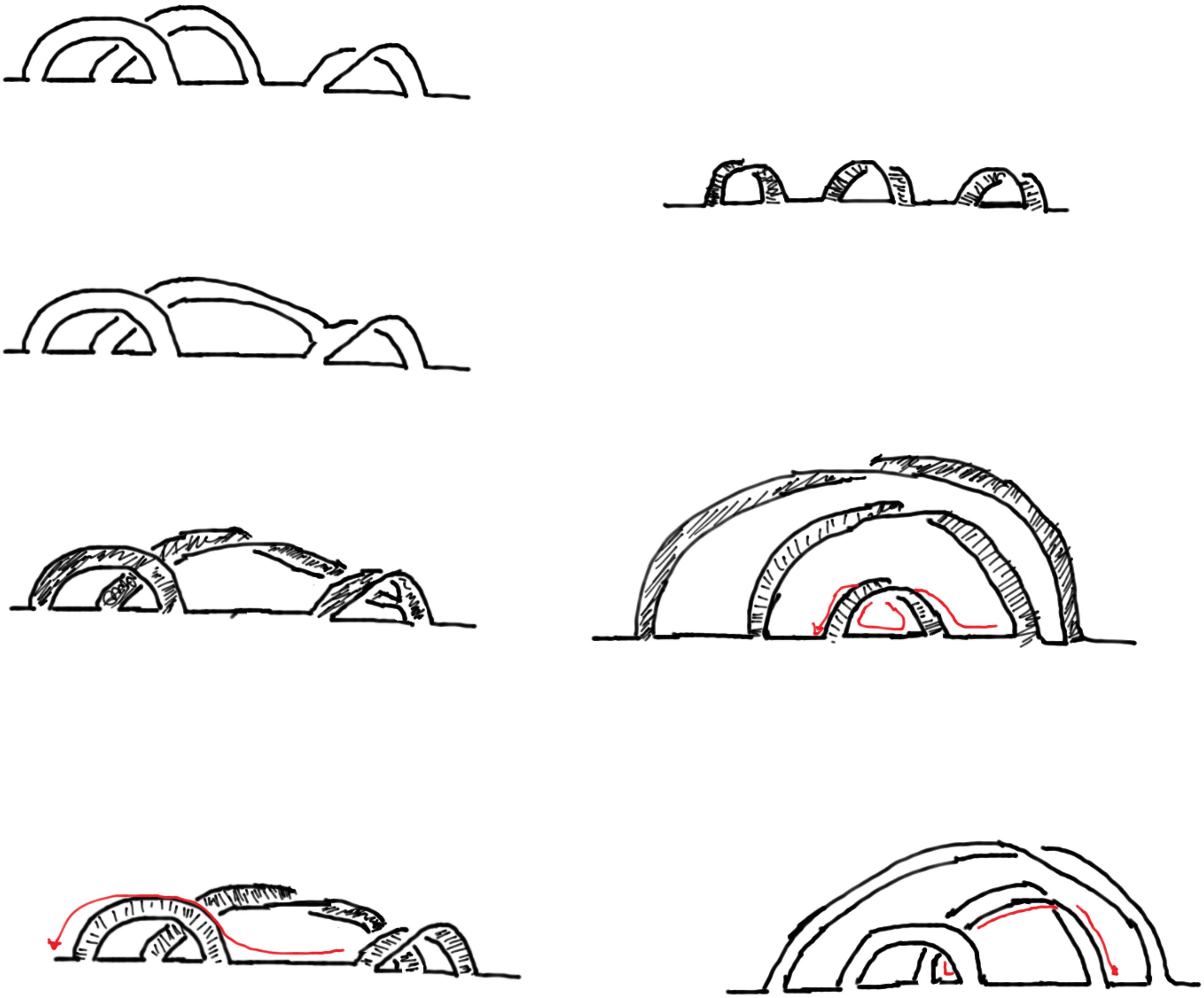
ONE ∂ comp.







$$T^2 \# P^2 = P^2 \# P^2 \# P^2$$