Fourth Homework

Due: Tuesday, February 28

1. Find a basis for the kernel and the range of each of the following linear transformations:
   (a) $T\colon \mathbb{R}^3 \to \mathbb{R}$ given by $T(\mathbf{x}) = (x + y + 2z, -x + y - z, -x + 3y)$, where $\mathbf{x} = (x, y, z)$.
   (b) $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ with matrix
   $$[T] = \begin{pmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & -3 & 3 \end{pmatrix}$$
   (c) $T\colon \mathbb{R}^4 \to R^3$ given by
   $$T(\mathbf{x}) = (2x + 9w + 4z + 6w, 5x + 22y + 9z + 14w, x + 4y + z + 2w)$$
   where $\mathbf{x} = (x, y, z, w)$.
   (d) $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ given by $T(\mathbf{x}) = (x - y, x + y, 2x - y)$, where $\mathbf{x} = (x, y)$.
   (e) The linear transformation $T\colon \mathbb{R}^3 \to \mathbb{R}^2$ defined by
   $$T(\mathbf{e}_1) = (2, -1), \quad T(\mathbf{e}_2) = (4, -2), \quad T(\mathbf{e}_3) = (1, 1)$$

2. The linear transformation $T\colon \mathbb{R}^4 \to \mathbb{R}^3$ with matrix
   $$[T] = \begin{pmatrix} 2 & 1 & 3 & -4 \\ 1 & 3 & 4 & 3 \\ -1 & 2 & 1 & 7 \end{pmatrix}$$
   is *not* surjective. Find a vector $\mathbf{b} \in \mathbb{R}^3$ that is *not* in the range of T.

3. For each of the following linear transformations T, determine whether T is an isomorphism. If it is, find T^{-1}. If it is not, find a basis of $\ker T$.

4. Find the inverse of the following matrices:
   (a) $\begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}$
   (b) $\begin{pmatrix} 1 & -1 & 2 \\ 1 & 0 & 4 \\ 0 & 1 & 1 \end{pmatrix}$

5. For each of the following systems, write it in the form $A\mathbf{x} = \mathbf{b}$ and solve it using A^{-1}.
   (a) $\begin{cases} 3x + y = 5 \\ x + y = 7 \end{cases}$
   (b) $\begin{cases} x - y + 2z = -6 \\ x + 4z = 6 \\ y + z = -8 \end{cases}$

6. Find the coordinates of the vector $\mathbf{v} = -2\mathbf{i} + 7\mathbf{j} - 3\mathbf{k}$ with respect to the basis $B = \{\mathbf{i} + \mathbf{k}, 2\mathbf{i} - \mathbf{j}, \mathbf{j} - 3\mathbf{k}\}$.