Third Homework

Due: Tuesday, February 28

1. Find a basis for the linear subspace of $\mathbb{R}^3$ spanned by the vectors $(0, 2, -1)$, $(1, 1, 1)$, and $(1, 3, 0)$.

2. In $\mathbb{R}^4$ let $B = \{(1, 0, 0, 0), (1, 1, 0, 0), (1, 1, 1, 0), (1, 1, 1, 1)$
   (a) Show that B is a basis of $\mathbb{R}^4$
   (b) Express every standard basic vector $\mathbf{e}_i$, $i = 1, 2, 3, 4$ as a linear combination of elements of B.

3. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be given by:
$$T(x\mathbf{i} + y\mathbf{j}) = (3x + 5y)\mathbf{i} + (8x - 3y)\mathbf{j} - 4x\mathbf{k}$$
   (a) Prove that T is a linear transformation.
   (b) Find $[T]$, the matrix of T.

4. Let $T\colon \mathbb{R}^4 \to \mathbb{R}^3$, and $S\colon \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformations defined by:
$$Tlr((x_1, x_2, x_3, x_4)) = (-x_1 + 3x_2 + x_3 + 9x_4, 2x_1 + x_3 + 7x_4, 4x_1 + 2x_2 + x_3 + 2x_4)$$
   and
$$S\left((x_1, x_2, x_3)\right) = (x_1 - 2x_3 + 3x_3, 5x_1 + 4x_2 + 2x_3)$$
   (a) Find the matrices $[T]$ and $[S]$.
   (b) Compute the product $[S][T]$.
   (c) Write a formula for $S \circ T$ using the answer in part (b).