Second Homework

Due: Tuesday, February 21

1. Solve the following systems. If the system is consistent give a parametric equation of the solution set.

   (a) $\begin{cases} 2x - 3y &= 6 \\ 4x - 6y &= 12 \end{cases}$

   (b) $\begin{cases} x + \ \ y &= 0 \\ 2x + 2y &= 1 \end{cases}$

   (c) $\begin{cases} x_1 - \ \ x_2 + \ \ x_3 &= 1 \\ 2x_1 + \qquad\ \ x_3 &= 3 \\ 5x_1 - 2x_2 + 3z &= 5 \end{cases}$

   (d) $\begin{cases} x + \ \ y + \ \ z &= 9 \\ 2x + 4y - 3z &= 1 \\ 3x + 6y - 5z &= 0 \end{cases}$

   (e) $\begin{cases} 2x - 2y &= 0 \\ \qquad\qquad z + 3w &= 2 \\ 3x - 3y &= 0 \\ \qquad\qquad 2z + 6w &= 4 \end{cases}$

   (f) $\begin{cases} x_1 + 2x_2 + 3x_3 &= 1 \\ 2x_1 - \ \ x_2 + \ \ x_3 &= 2 \\ x_1 - 8x_2 - 7x_3 &= 1 \\ \qquad x_2 + \ \ x_3 &= 0 \end{cases}$

2. In $\mathbf{R}^4$, let $\mathbf{a} = (2, -4, 3, 5)$, $\mathbf{b} = (1, -5, 1, -2)$, $\mathbf{c} = (0, 2, 1, 2)$, $\mathbf{v} = (12, -6, 23, 54)$, and $\mathbf{u} = (3, 2, -1, 4)$. Determine whether each of $\mathbf{v}$, $\mathbf{u}$ are in $\langle \mathbf{a}, \mathbf{b}, \mathbf{c} \rangle$. If yes then express the vector as a linear combination of $\mathbf{a}$ $\mathbf{b}$, and $\mathbf{c}$.

3. In $\mathbb{R}^3$, consider the set $B = \{(1, 1, 1), (1, 1, 0), (1, 0, -1)$.

   (a) Write $\mathbf{v} = (3, 1, 5)$ as a linear combination of vectors from B.

   (b) Show that *any* vector $\mathbf{b} = (b_1, b_2, b_3)$ can be written as a linear combination from vectors from B, in a *unique* way.