BRONX COMMUNITY COLLEGE
of the City University of New York

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

MATH 35 | Exam 1
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Directions: Write your answers in a separate piece of paper. Please staple all sheets together. To get full credit you *must* show all your work. Simplify your answers whenever possible. Be certain to indicate your final answer clearly.

1. Consider the 2×2 matrix
$$A = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$$
   (a) Find the eigenvalues and the corresponding eigenvectors of A.
   (b) Use the result of part a to solve the system of linear differential equations:
$$\begin{cases} y_1' &= 2y_1 + 2y_2 \\ y_2' &= \ \ y_1 + 3y_2 \end{cases}$$
   subject to the initial conditions $y_1'(0) = 2$, $y_2'(0) = 1$.

2. Let $f\colon \mathbb{R}^3 \to \mathbb{R}$ be given by $F(x, y, z) = x$. Find the maximum and minimum of f subject to the constrains
$$x^2 + y^2 + z^2 = 1 \quad \text{and} \quad x + y + z = 1$$

3. Show that $x^3z^2 - z^3yx = 0$ can be solved for z near $(1, 1, 1)$ but not near the origin. Compute $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ at $(1, 1)$.

4. Show that
$$\mathbf{c}(t) = \left(\frac{1}{1-t}, 0, \frac{e^t}{1-t}\right)$$
   is a flow line of the vector field $\mathbf{F}(x, y, z) = (x^2, 0, z(1 - x))$.

5. Find and graph the integral curve through the point $(1, 0)$ for each of the following vector fields.
   (a) $\mathbf{F}(x, y) = (4x + 2, x + 3y)$
   (b) $\mathbf{F}(x, y) = (-x + 2y, -x - y)$
   (c) $\mathbf{F}(x, y) = (x - 5y, x - y)$
   (d) $\mathbf{F}(x, y) = (-2x - y, x - 4y)$

6. Show that $\mathbf{F}(x, y) = (x^2 + x^2)\mathbf{i} - 2xy\mathbf{j}$ is *not* a gradient vector field.

7. Sketch or describe the region of integration for

$$\int_0^1 \int_0^x \int_0^y f(x, y, z)\, dz\, dy\, dx$$

and interchanges the order to $dy\, dx\, dz$.

8. Evaluate

$$\iiint_D e^{\left(x^2+y^2+z^2\right)^{3/2}}\, dx\, dy\, dz$$

where D is the region defined by $1 \le x^2 + y^2 + z^2 \le 2$ and $z \ge 0$.