BRONX COMMUNITY COLLEGE
of the City University of New York

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

MATH 35 — Exam 1
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Name: ____

Directions: Write your answers in the provided booklets. To get full credit you *must* show all your work. Simplify your answers whenever possible. Be certain to indicate your final answer clearly.

1. Find an equation for the plane in R^3 that is spanned by the vectors $\mathbf{a} = 2\,\mathbf{i} - 3\,\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = \mathbf{j} - 5\,\mathbf{k}$

2. In $\mathbb{R}^3$, let $S = \{(-1, 2, 1), (3, 1, 2), (1, 5, 4)\}$.
   (a) Find a basis for $\langle S \rangle$.
   (b) Let $\mathbf{w} = (-6, 5, 1)$, and $\mathbf{v} = (1, 1, 1)$. For each of these vectors determine whether it is in $\langle S \rangle$. If it is then write the vector as a linear combination of elements of S.

3. In $\mathbb{R}^3$ consider the vectors $\mathbf{u} = (-1, 3, -5)$, $\mathbf{v} = (2, -5, 9)$, and $\mathbf{w} = (3, 1, -2)$.
   (a) Prove that the set $B = \{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ forms a basis of $\mathbb{R}^3$.
   (b) Find the coordinates of the vector $\mathbf{a} = (-7, 14, -21)$ with respect to the basis B. In other words, express the vector $\mathbf{a}$ as a linear combination of elements of B.
   (c) Find the volume of the parallepiped generated by these three vectors.

4. Let $T\colon \mathbb{R}^3 \to \mathbb{R}^4$ be defined by
$$T(\mathbf{v}) = (2\,x + 5\,y + 3\,z, -3\,x + 2\,y + 5\,z, x - 3\,y - 4\,z, 4\,x - 4\,z)$$
   where $\mathbf{v} = (x, y, z) \in \mathbb{R}^3$.
   (a) Find $[T]$ the matrix of T.
   (b) Prove that T is neither injective nor surjective.
   (c) Find a basis for each of the subspaces $\operatorname{im} S$ and $\ker S$.

5. Let
$$A = \begin{pmatrix} 1 & -2 & 3 & -4 \\ 4 & -7 & 14 & -19 \\ 3 & -6 & 10 & 8 \\ 2 & -4 & 6 & -9 \end{pmatrix}$$
   (a) Find A^{-1}.
   (b) Use the answer in Part (a) to solve the system:
$$\begin{cases} x - 2y + \ \ 3z - 4w &= 0 \\ 4x - 7y + 14z - 19w &= 4 \\ 3x - 6y + 10z + \ \ 8w &= 5 \\ 2x - 4y + \ \ 6z - \ \ 9w &= 0 \end{cases}$$

6. Consider the following linear transformations: $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ given by
$$T(x, y) = (2x - y, 3x + 2y, -x + y)$$
   , and $S\colon \mathbb{R}^3 \to \mathbb{R}^2$ given by
$$S(x, y, z) = (x + y + z, -3x + 2y + 4z)$$
   .

(a) Find the matrices $[T]$ and $[S]$.

(b) Find the products $[S]\,[T]$, and $[T]\,[S]$

(c) Use the answer to Part (b) to write down a formula for the transformations $S \circ T$ and $T \circ S$.

(d) Find the rank and the nullity of $S \circ T$ and $T \circ S$.

7. **Extra Credit:** Find the eigenvalues and bases for the corresponding eigenspaces for the following 2×2 matrix

$$A = \begin{pmatrix} -2 & 4 \\ 4 & 4 \end{pmatrix}$$