Spring 2017

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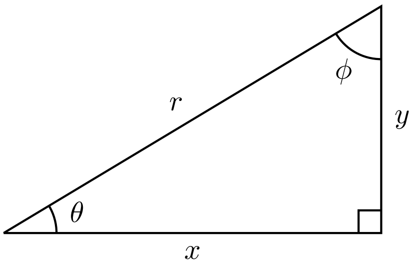
1 Applications of Right Triangle Trigonometry

Last time we saw: Consider the following right triangle.



We have the following formulas for the trigonometric ratios of the two acute angles θ and ϕ:

$$\sin\theta = \frac{y}{r} \quad \cos\theta = \frac{x}{r} \quad \tan\theta = \frac{y}{x}$$
$$\sin\phi = \frac{x}{r} \quad \cos\phi = \frac{y}{r} \quad \tan\phi = \frac{x}{y}$$

These formulas can be solved for any of the sides. For example if we know r and the angle θ we can find x and y:

$$x = r\cos\theta, \quad y = r\sin\theta$$

If we know x and θ we can find y and r:

$$y = x\tan\theta, \quad r = \frac{x}{\cos\theta}$$

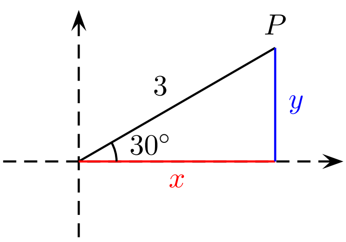
If we know y and θ we can find x and r:

$$x = \frac{y}{\tan\theta}, \quad r = \frac{y}{\sin\theta}$$

Today: We will see various applications of these ideas.

Example 1. Find the coordinates of the point P if it is 3 units away from the origin $(0, 0)$ and the line OP forms an angle of $30°$ with the x-axis.

Answer.



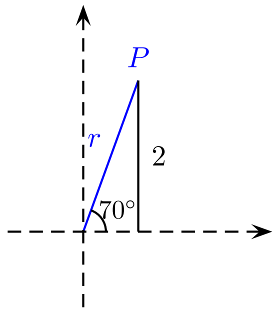
We have:

$$x = 3\cos 30° = \frac{3\sqrt{3}}{2} \qquad y = 3\sin 30° = \frac{3}{2}$$

□

Example 2. What is the distance of the point P from the origin, if its y-coordinate is 2, and the line OP forms an angle of $70°$ with the x-axis

Answer.



We have:

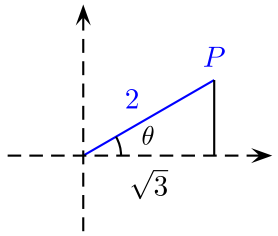
$$r = \frac{2}{\sin 70°} \approx \frac{2}{0.939692620786} = 2.12835554495$$

So $r \approx 2.13$.

□

Example 3. A point P has x coordinate $\sqrt{3}$ and is at distance 2 from the origin. Find the angle that OP forms with the x-axis.

Answer.



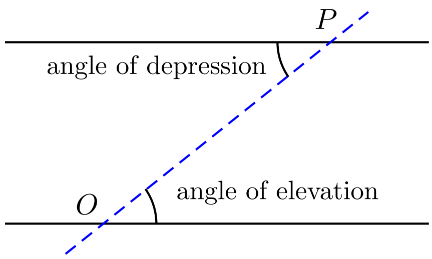
We have:

$$\cos\theta = \frac{\sqrt{3}}{2}$$

We know that $\cos^{-1}\frac{\sqrt{3}}{2} = 30°$. Therefore $\theta = 30°$.

□

Often in applications, we see the terms *angle of elevation* and *angle of depression*.

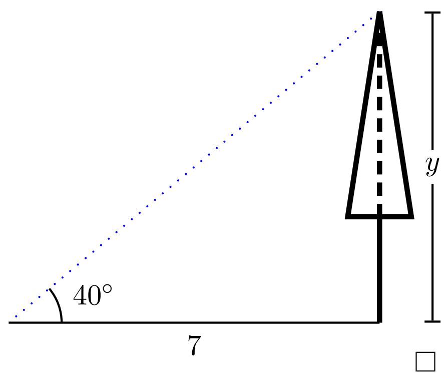


Notice that the angle of elevation from O to P equals the angle of depression from P to O.

Example 4. Find the height of a tree, if it's 7 feet away from where you're standing, and the *angle of elevation* from the ground to its highest point is $40°$.

Answer. The angle of elevation is the angle formed with the horizontal line. So we have the following picture:

If y is the height of the tree, we have $y = 7\tan 40°$. Now $\tan 40° \approx 0.839099631177$, therefore $y \approx 7 \cdot 0,84 = 5.88$. So the tree is approximately 5.88 feet high.



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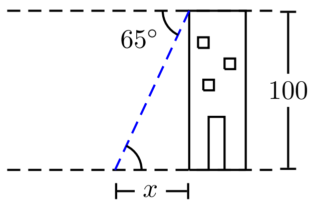
Example 5. You're sitting at the top of a 100 meter tall building and you look at your friend in the ground with an *angle of depression* of 65°. How far is your friend from the base of the building?

Answer. The angle of depression from your point of view is the same as the angle of elevation from your friend's point of view.

If x is the distance from the base of the building then $x = \frac{100}{\tan 65°}$. Calculator says: $\tan 65° \approx 2.14$. So

$$x = \frac{100}{2.14} \approx 46.635$$

So your friend is approximately 46.63 feet away from the base of the building.



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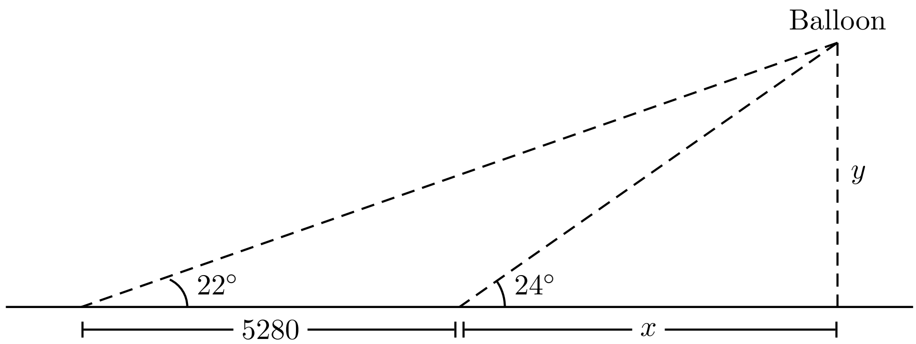
In some problems, we know the angles of elevation (or depression) from two different points, and the distance between those two points.

Example 6. A hot-air balloon is floating above a straight road. To calculate their height above the ground, the balloonists simultaneously measure the angle of depression to two consecutive mileposts on the road on the same side of the balloon. The angles of depression are found to be 22° and 24°. How high (in feet) is the balloon?

Answer. The distance between two consecutive mileposts is 1 mile, or since we want our answer in feet, 5280 feet. We have the following figure (not drawn to scale):



We have $\sin 22° \approx 0.37$ and $\sin 24° \approx 0.41$. If the first milestone is in horizontal distance x feet from the balloon the second milestone is $x + 5280$ feet away. So we can express the height y in two different ways.

Using the first point we get

$$y = x \sin 24° \approx 0.41x$$

and from the second we get

$$y = (x + 5280) \sin 22° \approx (x + 5280)0.37 = 0.37x + 1953.6$$

So we get the equation:

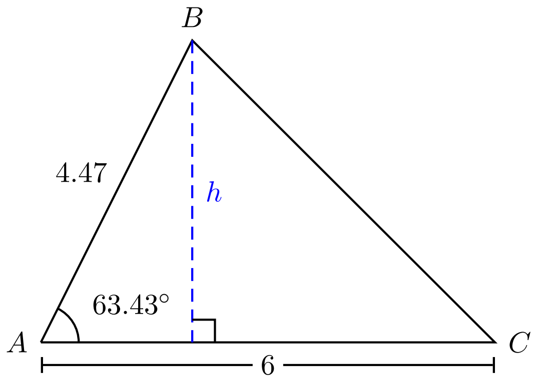
$$0.41x = 0.35x + 1953.6 \iff 0.04x = 1953.6 \iff x = \frac{1953.6}{0.4} = 48840$$

So we can calculate

$$y = 0.41 \cdot 48840 = 20024.4$$

So the balloon is approximately at a height of $20,024$ feet. $\square$

Example 7. Find the area of the triangle:



Answer. Recall that the area of a triangle is given by the formula

$$A = \frac{1}{2} b\, h$$

where b stands for the *base* of the triangle, in our case $b = 6$, and h, stands for the *height* to the base.

In our case we know the base, and we need to calculate the height. From the figure we see that $h = 4.47 \sin 63.43°$. Calculator says $\sin 63.43° \approx 0.89$. So $h \approx 4$.

So the area of the triangle is approximately:

$$A \approx \frac{1}{2} 6 \cdot 4 = 12$$

$\square$

In general we have the following formula: If in a triangle ABC the area is

$$\boxed{A = \frac{1}{2}\, b\, c\, \sin A}$$