

Simplifying Radical Expressions

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1 Radical expressions

1. For which real values of the variable x are the following radical expressions defined as real numbers?

- (a) \sqrt{x}
- (b) $\sqrt{-x}$
- (c) $\sqrt{3x-2}$
- (d) $\sqrt{6-2x}$
- (e) $\sqrt{3(x-5)+x}$
- (f) $\sqrt{x^2}$
- (g) $\sqrt{x^2+1}$
- (h) $\sqrt{-x^2}$
- (i) $\sqrt{x^2-4}$
- (j) $\sqrt{x^2+2x+1}$

2. Simplify each of the following radical expressions if the variables represent arbitrary real numbers. Can you simplify further if you assume that all the variables represent non negative numbers?

- (a) $\sqrt{z^2}$
- (b) $\sqrt{y^8}$
- (c) $\sqrt{x^3}$
- (d) $\sqrt{4x^2y^4}$
- (e) $\sqrt{50x^3y^5z^7}$
- (f) $\sqrt{\frac{48x^5y^2}{81z^4}}$
- (g) $\sqrt{\frac{72x^4y^3}{45z^8}}$

3. Simplify each of the following expressions. Assume all variables represent positive numbers.

- (a) $\sqrt[3]{8x^5y^6}$
- (b) $\sqrt[4]{32x^4y^5z^{13}}$
- (c) $\sqrt{\sqrt[3]{x}}$
- (d) $\sqrt{\sqrt{x^2z^6}}$
- (e) $\sqrt{50\sqrt{4xy^4}}$

4. Rationalize the denominator. Assume that all expressions are defined.

- (a) $\frac{3}{4+\sqrt{2}}$
- (b) $\frac{\sqrt{3}}{\sqrt{3}-5}$
- (c) $\frac{2\sqrt{6}}{4-\sqrt{2}}$

(d) $\frac{1}{\sqrt{3} - \sqrt{5}}$

(e) $\frac{\sqrt{10}}{2\sqrt{5} + 3\sqrt{2}}$

(f) $\frac{3}{1 - \sqrt{x}}$

(g) $\frac{\sqrt{30x}}{x - 2\sqrt{3}}$

(h) $\frac{\sqrt{a} - \sqrt{b}}{\sqrt{a} + \sqrt{b}}$

(i) $\frac{h}{\sqrt{x+h} - \sqrt{x}}$

5. Verify that:

(a) $\sqrt{11 - 6\sqrt{2}} = 3 - \sqrt{2}$

(b) $\sqrt{21 - 4\sqrt{5}} = 2\sqrt{5} - 1$

6. Verify that $x = 2 - \sqrt{3}$ is a solution to the following equation:

$$x^2 - 4x + 1 = 0$$

7. Verify that $3 + \sqrt{2}$ is a solution to the equation

$$x^3 - 7x^2 + 13x - 7 = 0$$