Review for the First Exam

Spring, 20017
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1. The larger leg of a right triangle is 3 inches more the smaller. The hypotenuse is $3\sqrt{5}$ inches. Find the length of the two legs.

2. For which real values of the variable x is the expression $\sqrt{4-2x}$ defined as a real number?

3. Simplify:
   (a) $\sqrt{27} - 5\sqrt{12} + 7\sqrt{3}$
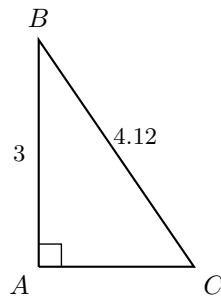   (b) $(2+\sqrt{6})(\sqrt{2}-\sqrt{15})$
   (c) $(2+\sqrt{(5)})^2$

4. Rationalize the denominator: $\dfrac{\sqrt{3}-2\sqrt{5}}{\sqrt{2}+\sqrt{3}}$

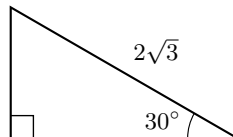5. Simplify assuming that all variables represent non-negative real numbers:
   (a) $\sqrt{75x^5y^7z^8}$
   (b) $\sqrt{\dfrac{75x^2y^5}{64z^8}}$

6. Find the measure of the angle B:

7. The angle of elevation of the the top of a building, measured 100 feet from its base is $25°$. How high is the building?
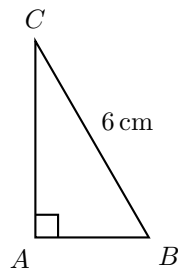
8. Find the area of the following right triangle:

9. Solve: $\sqrt{5x+6} = x$

10. Solve: $\sqrt{x+1} = 1 - \sqrt{2x}$

11. Given that $B = 60°$, solve the following triangle:

$A = 90°$ $\qquad a = 6$ cm

$B = 60°$ $\qquad b =$

$C =$ $\qquad c =$
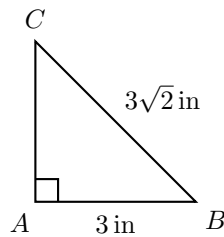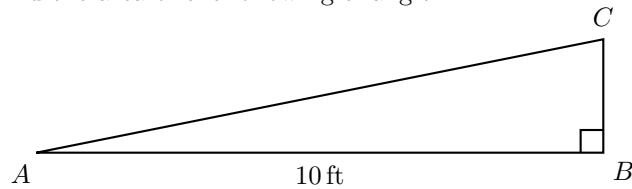

12. Solve the following triangle:

$A = 90°$ $\qquad a = 3\sqrt{2}$ in

$B =$ $\qquad b =$

$C =$ $\qquad c = 3$ in

13. Given that $A = 11.31°$, find the area of the following triangle:

14. Solve:

   (a) $\sqrt{x-3} = \sqrt{x} - 1$

   (b) $\sqrt{x-2} + 8 = x$

   (c) $\sqrt{x+20} - 2x = -5$

   (d) $\sqrt{2x} = \sqrt{x-4} + 2$

15. Solve:

   (a) $2x^2 - 3 = 6x$

   (b) $x^2 - 4x = 41$

   (c) $x^2 = 2x + 35$

   (d) $3x^2 - 10 = 2x$

   (e) $7x^2 + 2x - 16 = x^2 + x - 1$

16. An observer stands on level ground at a distance of 100 feet from the base of a building. How high, to the nearest foot, is the building if the angle of elevation is $25°$?
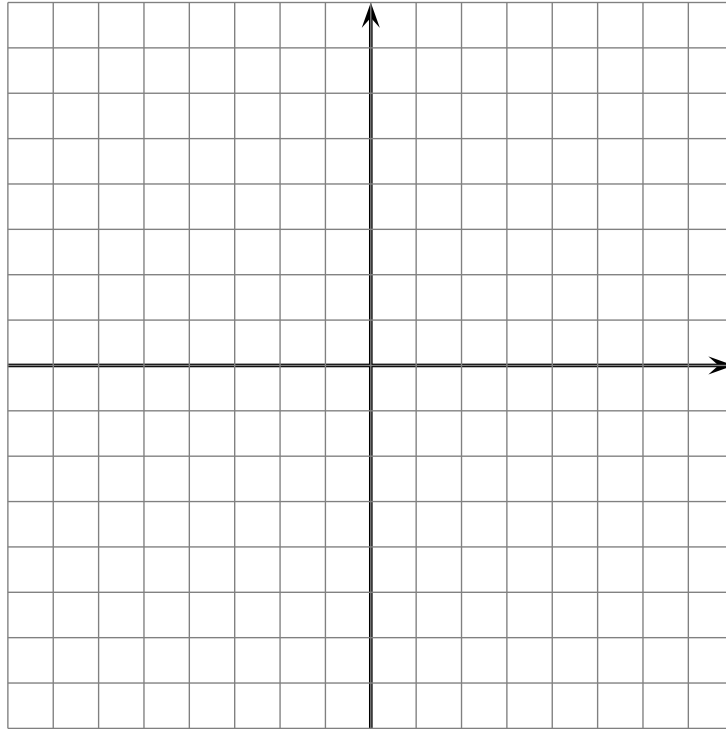
17. Simplify:

   (a) $5\sqrt{12} - \sqrt{200} + 3\sqrt{18}$

   (b) $\left(1 - \sqrt{3}\right)^2$

18. Find the exact value of the expression $\csc 60° \left(\cos 60° + \sin 45°\right)$. Simplify your answer as much as possible.

19. Graph the parabola $y = x^2 - 6x + 4$. Your graph should correctly indicate the vertex, the axis of symmetry, the x-intercepts, the y-intercept and the point symmetric to the y-intercept.

20. Graph the parabola $y = -2x^2 - 4x + 3$. Your graph should correctly indicate the vertex, the axis of symmetry, the x-intercepts, the y-intercept and the point symmetric to the y-intercept.
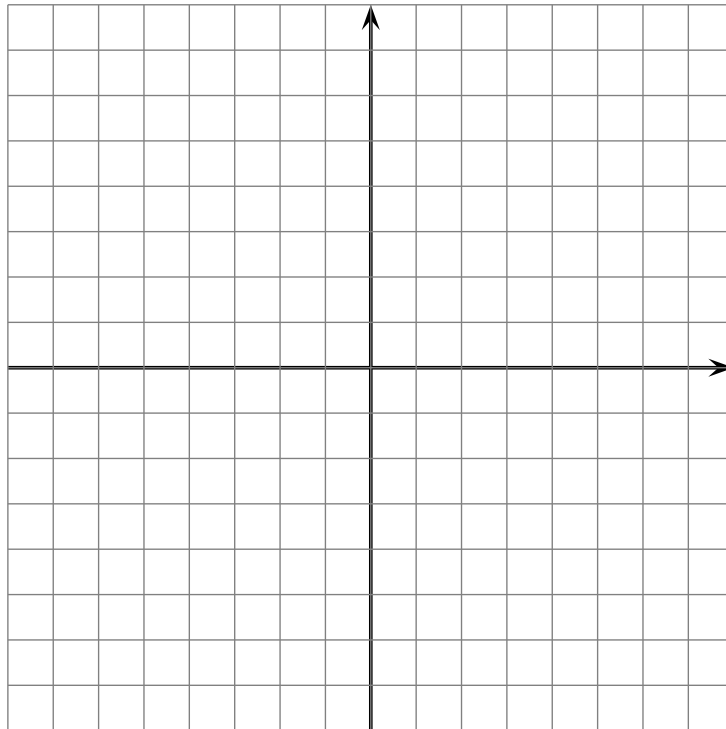
21. The angle of elevation of the top of a building, measured 200 feet from its base is 14.04°. What's the height of the building?

22. A boat is observed from the top of a lighthouse, 250 feet above sea level. If the boat is 2500 feet away what's the angle of depression?

23. Simplify assuming all variables represent positive numbers. The answer should contain only positive integers as exponents.
$$\left(\frac{27x^{15}y^{-\frac{21}{2}}}{8z^{-\frac{3}{2}}}\right)^{-\frac{1}{3}}$$

24. The length of one leg of a right triangle is one unit more that the length of the other leg. The length of the hypotenuse is $\sqrt{41}$ units.

   (a) Find the lengths of the two legs.
   (b) Find the measure of the two acute angles of the triangle.

25. The angle of elevation of the top of a building taken 300 feet from the base of the building is 22°. Find the height of the building to the nearest foot.

26. Simplify: $5\sqrt{52} - 3\sqrt{60} + 2\sqrt{13} + 3\sqrt{135}$

27. Simplify: $\dfrac{2\sqrt{5} - 5\sqrt{2}}{\sqrt{10} - 2}$

28. Simplify assuming all variables represent positive numbers. The answer should contain only positive integers as exponents.
$$\left(\frac{27x^{15}y^{-\frac{21}{2}}}{8z^{-\frac{3}{2}}}\right)^{-\frac{1}{3}}$$

29. Solve: $\sqrt{x-5} - \sqrt{x-1} = 3$

30. Simplify: $\sqrt{500} - 3\sqrt{20} - 2\sqrt{45} + 2\sqrt{5}$.

31. Simplify assuming all variables represent positive numbers. The answer should contain only positive integers as exponents.
$$\left(\frac{4x^{-8}y^3}{z^{-6}}\right)^{-\frac{1}{2}}$$

32. Solve $x = \sqrt{x+7} - 5$.

33. One leg of a right triangle is $2\,\text{cm}$ more than the other. If the hypotenuse is $\sqrt{7}\,\text{cm}$

   (a) find the lengths of the two legs.
   (b) Find the measures of the two acute angles of the triangle.

34. Simplify: $3\sqrt{28} - \sqrt{700} + 4\sqrt{63}$

35. Simplify: $\dfrac{\left(3 - \sqrt{2}\right)^2}{1 + \sqrt{2}}$

36. Simplify, assuming all variables represent positive numbers: $\sqrt{\dfrac{9b^8c^3}{20a^7}}$
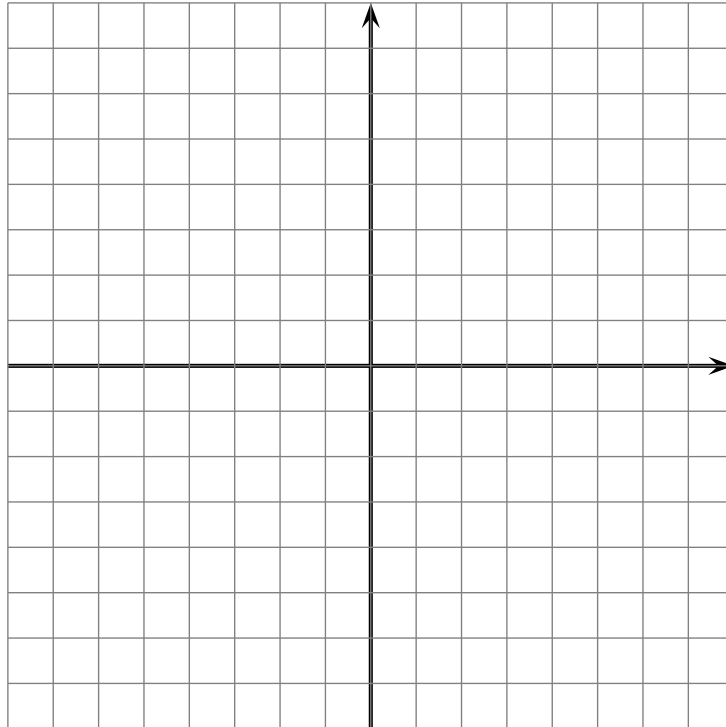
37. Simplify assuming all variables represent positive numbers. The answer should contain only positive integers as exponents.

$$\left(\frac{x^{21}y^{-\frac{15}{4}}}{z^{-\frac{9}{2}}}\right)^{-\frac{2}{3}}$$

38. Solve: $x - \sqrt{x-4} = 10$

39. Solve: $\sqrt{x} - \sqrt{x-9} = 1$

40. Sketch a graph of the parabola $y = x^2 - 6x + 5$. The graph should correctly indicate the vertex and the axis of symmetry.

41. Evaluate (give *exact* answer): $\dfrac{\cot 30°}{2 - \tan 45°}$

42. Solve: $x^2 - 2x + 2 = 0$

43. In a triangle ABC we have $A = 90°$, $a = \sqrt{15}$, and $b = \sqrt{7}$. Find c.

44. One leg of a right triangle is $5\,\text{cm}$ and the hypotenuse is $10\,\text{cm}$. Find the angles of the triangle.

45. The sum of the lengths of the legs of a right triangle is $4\,\text{cm}$. The hypotenuse is $3\sqrt{2}$ cm long. Find the lengths of the two legs.

46. Simplify: $2\sqrt{63} + 2\sqrt{28} - \sqrt{700}$

47. Simplify: $(\sqrt{6} - 5)(\sqrt{2} + \sqrt{3})$

48. Simplify: $49 - 12\sqrt{5} - (2 - 3\sqrt{5})^2$

49. Simplify, assuming all variables represent positive numbers: $\sqrt{\dfrac{12x^7y^2}{25z^4}}$

50. Solve: $\sqrt{x+4} - 2x = -7$

51. Solve: $\sqrt{x+6} + \sqrt{7-x} = 5$

52. Simplify assuming all variables represent positive numbers. The answer should contain only positive integers as exponents.

$$\left(\frac{x^{-6}y^4}{z^{\frac{3}{2}}}\right)^{\frac{2}{3}}$$

53. Find the length r and the angle θ.