BRONX COMMUNITY COLLEGE
of the City University of New York

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

MATH 42 | Midterm
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Name: ____

Instructions:

- Write your answers in the provided booklets.**Show all your work**

1. Solve the following system.

$$\begin{cases} x_1 - x_2 + 2x_3 - x_4 &= -1 \\ 2x_1 + x_2 - 2x_3 - 2x_4 &= -2 \\ -x_1 + 2x_2 - 4x_3 + x_4 &= 1 \\ 3x_1 \qquad\qquad\qquad - 3x_4 &= -3 \end{cases}$$

2. Verify that the indexed set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly dependent and express one of the vectors as a linear combination of the other vectors.

$$\mathbf{v}_1 = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} -10 \\ 2 \\ -4 \end{bmatrix}$$

3. A transformation $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ is defined by $T(x, y, z) = (2x - y + 3z, 4z, 2y - 5z)$.
   (a) Prove that T is linear.
   (b) Find the standard matrix of T.
   (c) Find the volume of $T(S)$ where S is the parallepiped determined by the vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, where:

$$\mathbf{v}_1 = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$$

4. Let

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & 0 \\ -2 & -1 & 1 \end{bmatrix}$$

   (a) Find A^{-1}

   (b) Use the result in part (a) to solve the system $A\mathbf{x} = \mathbf{b}$ where

$$\mathbf{b} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

5. Prove *one* of the following:

   (a) If A is a singular $n \times n$ matrix then there exists a non-zero $n \times n$ matrix B such that $A \cdot B = O$.

   (b) If A is invertible then $\det A \neq 0$.

6. Mark each of the following statements as **T**rue or **F**alse. Justify your answer.

   (a) Let A be an $m \times n$ matrix. If the homogeneous system $A\mathbf{x} = \mathbf{0}$ has non-trivial solutions then for some $\mathbf{b} \in \mathbb{R}^m$ the system $A\mathbf{x} = \mathbf{b}$ is inconsistent.

   (b) Let A be an $m \times n$ matrix. If the columns of A span $\mathbf{R}^m$ then the linear transformation determined by A is one-to-one.

   (c) Let A be an $n \times n$ matrix. If the columns of A are linearly independent then the linear transformation determined by A is onto.

   (d) If A and B are $n \times n$ matrices then $\det(A \cdot B) = \det A \cdot \det B$

7. **Extra Credit** Let

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

   Find all 2×2 matrices B that commute with A, i.e. they satisfy $AB = BA$.