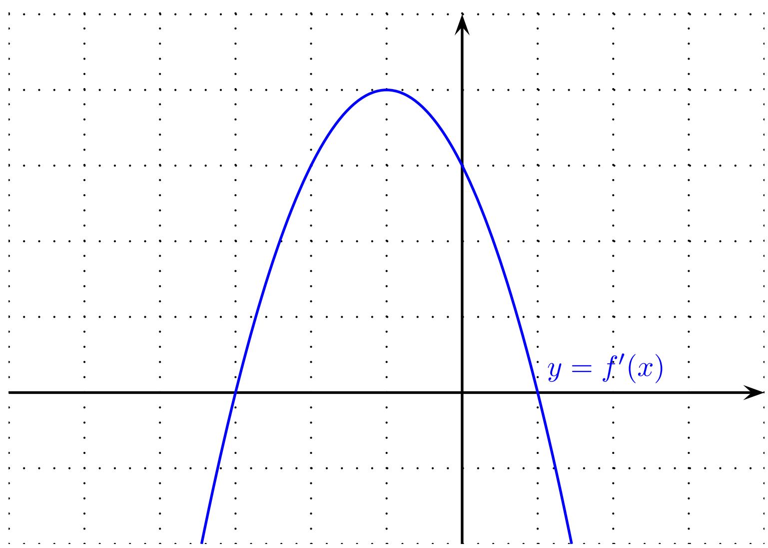
Review Questions for the Second Exam

1. Consider the function $f(x) = \sin x$.
   (a) Verify that f satisfies the hypotheses of Rolle's theorem on the interval $[0, \pi]$.
   (b) Find all c that satisfy the conclusion of Rolle's theorem.

2. Consider the function $f(x) = x^3 - x^2 - 4x + 11$.
   (a) Verify that f satisfies the hypotheses of Rolle's theorem on the interval $[-2, 2]$.
   (b) Find all c that satisfy the conclusion of Rolle's theorem.

3. Prove that the equation
$$x^3 - 6x^2 + 15x + 6 = 0$$
   has exactly one real solution.

4. The function f has the property that $f(0) = 1$, while the graph of its derivative is shown in the following picture:



   Draw a rough but accurate graph of f.

5. Sketch a qualitatively accurate graph of:
   (a) $y = \dfrac{4+6x}{\sqrt{x}}$
   (b) $y = \cot x$
   (c) $y = \sec x$

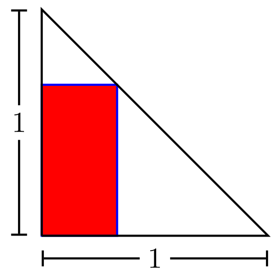
6. Let $f(x) = x^4 - 5x^2 + 4$.
   (a) Sketch a qualitatively accurate graph of $y = f(x)$.
   (b) Sketch a qualitatively accurate graph of $y = |f(x)|$.
   (c) Which one is larger $f(1.4567898765687654987)$ or $f(1.4567898765687654985)$?

7. Draw a qualitatively accurate graph of

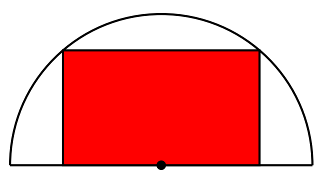
$$f(x) = \frac{x^2+1}{x^2-1}$$

The graph should correctly indicate any x and y intercepts, possible asymptotes, extrema and inflection points, the intervals where the function is increasing or decreasing, and the intervals where the function is concave upward or downward.

8. Consider a isosceles right triangle with legs of length 1 and a rectangle inscribed inside it in such a way that its sides are parallel to the legs of the triangle, as shown below. Find the dimensions of the rectangle so that its area is the maximum possible.



9. A rectangle is inscribed in a semicircle of radius 2 as shown below. What is the largest area among all such rectangles?



10. Calculate the following limits:
    (a) $\lim_{x\to-\infty}\left(\sqrt{4x^2+3}+2x\right)$
    (b) $\lim_{x\to\infty}\frac{3x^2-5x+3}{-5x^2-7x+8}$
    (c) $\lim_{x\to-\infty}\frac{2x^5-5x^4+3x-2}{5x^4+3x^2-2}$
    (d) $\lim_{x\to\infty}\frac{-5x^2+3x+11}{x+7}$
    (e) $\lim_{x\to\infty}\frac{\cos x}{x^2}$

11. Use Newton's method to approximate $\sqrt[3]{7}$ to five decimal places.

12. Use Newton's method to estimate $\sqrt{5}$. Your estimate should be correct to four decimal digits.

13. A particle moves in a line with velocity $v(t) = -3\sin 2t$. For the time interval $[0, \pi]$, find:

(a) the displacement of the particle,

(b) the total distance traveled by the particle.

(c) if the initial position of the particle is $s(0) = 3$, find the law of motion, that is an equation for $s(t)$.

14. A particle is moving on a line with velocity given by $v(t) = t^4 - 5t^2 + 4$.

    (a) Find the displacement of the particle for the time interval $0 \leq t \leq 2$.

    (b) Find the total distance covered by the particle for the time interval $0 \leq t \leq 2$.

    (c) Find the displacement and the total distance covered for the time interval $0 \leq t \leq 1$.

    (d) If at $t = 0$ the particle was at $s(0) = -4$, find the law of motion for this particle (i.e. a formula for $s(t)$).

15. Assume that the acceleration due to gravity near the surface of the earth is $g = 9.8\,\mathrm{m/s^2}$ and that there are no forces due to friction, resistance of the air, etc. From the edge of a $50\,\mathrm{m}$ building, a ball is thrown upwards with initial velocity of $5\,\mathrm{m/s}$.

    (a) Find the law of motion for the ball, i.e. an expression for $s = s(t)$.

    (b) When will the ball hit the ground?

    (c) What's the highest point that the ball will reach and when will it reach it?

    (d) How much is the total distance that the ball will cover until it hits the ground?