

BRONX COMMUNITY COLLEGE
of the City University of New York

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

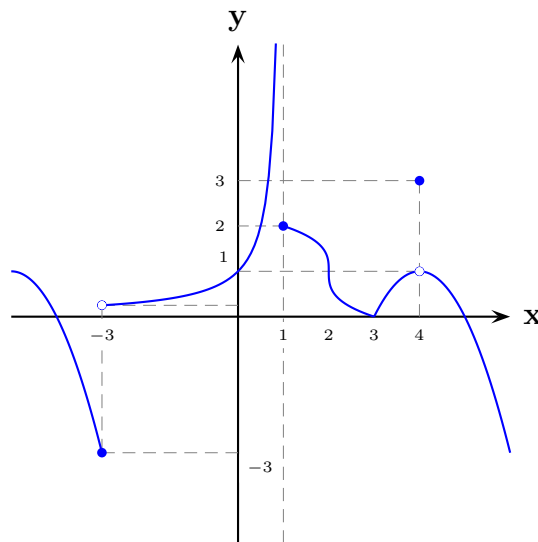
MATH 31
Nikos Apostolakis

Exam 1
October 15, 2018

Name: _____

Directions: Write your answers in the provided booklets. Make sure to indicate which answer belongs to which question. To get full credit you *must* show all your work. Simplify your answers whenever possible. Be certain to indicate your final answer clearly. This exam has a total of 1150 points. The perfect score for this exam is 1000 points.

1. The graph of $y = f(x)$ is shown bellow:



- (a) (50 points) At which points is f discontinuous? What is the nature of discontinuity at each of these points?
- (b) (50 points) At which points does f fail to be differentiable? Give reasons.
2. Find the following limits. Your answer should be a real number, $+\infty$, $-\infty$, or *Does Not Exist*.

(a) (25 points) $\lim_{x \rightarrow -5} \frac{x^2 - 2x - 35}{x + 5}$

(b) (25 points) $\lim_{x \rightarrow 0} \frac{\sin 5x}{3x}$

(c) (25 points) $\lim_{x \rightarrow -7} \frac{|x + 7|}{x + 7}$

(d) (25 points) $\lim_{x \rightarrow \frac{\pi}{2}^+} \tan x$

3. (100 points) Prove that the equation $5x^3 - 7x^2 + 8x - 1 = 0$ has a solution in the interval $(0, 1)$. Name any theorems you're using.

4. Let $f(x) = |x^2 - 2x - 3|$.

(a) (50 points) Sketch a graph of $y = f(x)$.

(b) (50 points) At what points f fails to be differentiable?

(c) (50 points) Find $f'(x)$ where it exists.

5. (100 points) Calculate $\frac{d}{dx}(x^2 - 5x)$ using the definition of the derivative as a limit of the difference quotient.

6. Calculate the following derivatives. Simplify your answer as much as possible:

(a) (25 points) $\left(\frac{x}{x-1}\right)'$

(b) (35 points) $(\sqrt{x} \cos \sqrt{x})'$

(c) (40 points) $\left(\sqrt[5]{x \tan x}\right)'$

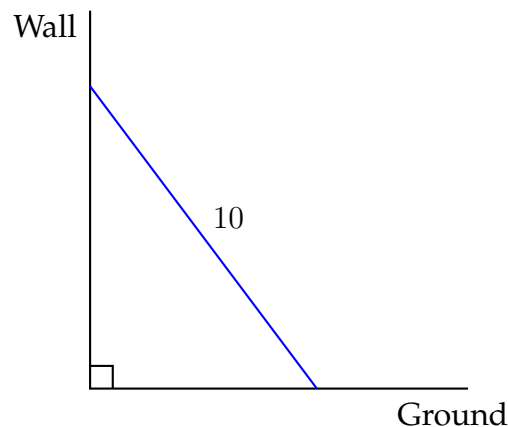
7. Consider the curve:

$$y^3 + x^3 = 2xy^2 + x - 1$$

(a) (100 points) Find an equation for y' , at the points that this equation can be solved to express y as a function of x .

(b) (50 points) Find the equation of the line tangent to the curve at the point $(-2, -1)$.

8. (150 points) A ladder 10 ft long rests against a vertical wall as in the figure bellow.



The bottom of the ladder slides away from the wall at a rate of 2 ft/s. How fast is the top of the ladder sliding down when the bottom of the ladder is 8 ft away from the wall?

9. A particle moves on a horizontal line according to the law of motion

$$s(t) = t^3 - 6t^2 + 9t + 5, \quad t \geq 0$$

where t is measured in seconds and s in meters.

- (a) (50 points) Find the velocity and acceleration of the particle as functions of time.
- (b) (25 points) When is the particle moving forward and when is it moving backwards?
- (c) (25 points) When is the particle speeding up and when is it slowing down?
- (d) (25 points) Find the total distance traveled by the particle in the first four seconds.

10. Let $f(x) = \sqrt[3]{x - 2}$.

- (a) (50 points) Find the linearization of f at $a = 10$
- (b) (25 points) Use this linearization to estimate $\sqrt[3]{7.98}$.