Review Questions

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1. Evaluate the given expression for the given values of the variables:
   (a) $x^2 - 3xy + 4xy^2$; $x = -2, y = 3$
   (b) $x^2 - 3xy + 4xy^2$; $x = -2, y = 2i$
   (c) $x^2 - 3xy + 4xy^2$; $x = \sqrt{2}, y = \sqrt{10}$
   (d) $\sqrt{b^2 - 4ac}$; $a = 1, b = 5, c = 6$
   (e) $\sqrt{b^2 - 4ac}$; $a = 3, b = -6, c = -4$
   (f) $\sqrt{b^2 - 4ac}$; $a = 12, b = -8, c = -15$
   (g) $\sqrt{b^2 - 4ac}$; $a = 2, b = 2, c = -3$
   (h) $\sqrt{b^2 - 4ac}$; $a = 1, b = -6, c = 34$
   (i) $\dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$; $a = 2, b = 5, c = -3$

2. Simplify as much as possible:
   (a) $\sqrt{80}$
   (b) $\sqrt{-90}$
   (c) $3\sqrt{45} - 2\sqrt{54} - 5\sqrt{20}$
   (d) $6\sqrt{600} + 7\sqrt{24} - 3\sqrt{48}$
   (e) $(1 - 2\sqrt{5})(1 + 2\sqrt{5}))$
   (f) $\sqrt{7}(\sqrt{14} - \sqrt{21})$
   (g) $(\sqrt{2} + \sqrt{(3)})(\sqrt{6} - \sqrt{10})$
   (h) $(2 + \sqrt{3})^2$
   (i) $(2 + \sqrt{3})^3$
   (j) $(2 + \sqrt{3})^4$
   (k) $(3 - i)(4 + 5i)$
   (l) $(5 - 6i)(5 + 6i)$
   (m) $(3 - 4i)^2$

3. Simplify, as much as possible. The final answer should not contain any radicals in the denominators.
   (a) $\dfrac{3}{\sqrt{6}}$
   (b) $\dfrac{2\sqrt{5}}{\sqrt{3}}$
   (c) $\dfrac{\sqrt{5}\sqrt{35}}{\sqrt{7}}$
   (d) $\dfrac{\sqrt{8}\sqrt{6}}{\sqrt{15}}$

4. Verify that the given value is a solution to the given equation:
   (a) $x^2 - 5x + 6 = 0$; $x = 2$
   (b) $12x^2 - 8x = 15$; $x = \frac{1}{2}$
   (c) $x^2 - 6x + 13 = 0$; $x = 3 + \sqrt{2}$
   (d) $x^2 + 8x = -41$; $x = -4 + 5i$
   (e) $x^4 - 14x^2 = -1$; $x = 2 + \sqrt{3}$
   (f) $x^4 - 16x^2 + 4 = 0$; $x = \sqrt{3} + \sqrt{5}$
5. Consider the quadratic polynomial:
$$x^2 + 8x + 10$$
   (a) Show that this polynomial cannot be factored using whole numbers.
   (b) However this polynomial can be factored using irrational numbers. Show by doing the calculations that:
$$(4 + \sqrt{6}) + (4 - \sqrt{6}) = 8$$
   and
$$(4 + \sqrt{6})(4 - \sqrt{6}) = 10$$
   So conclude that
$$x^2 + 8x + 10 = (x + 4 + \sqrt{6})(x + 4 - \sqrt{6})$$
   (c) Verify this factoring by multiplying the two factors in the right hand side.