

Review for the fourth exam

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Disclaimer The following is a set of questions to help you review for the coming fourth exam. You can write up the answers to these questions and bring them in, Monday, November 14, for extra credit.

As usual if you know how to answer these questions you should do fine in the exam, but there is no guarantee that the actual questions in the exam will be perceived to be similar to the following.

1. Given $a = -4$ and $b = 2$, evaluate the expression given below.

$$\begin{aligned} a^2 + 2ab - ab^2 &= (-4)^2 + 2(-4)(2) - (-4)(2)^2 \\ &= 16 + 2(-4)(2) - (-4)(4) \\ &= 16 - 16 + 16 \\ &= 16 \end{aligned}$$

- A. -48 B. -16 **C. 16** D. 48

2. Solve for x :

LCM = 6

$$\begin{aligned} \frac{3(x+4)}{2} &= \frac{2(x+9)}{3} \Leftrightarrow 3x+12 = 2x+18 \\ \Leftrightarrow 3x-2x &= 18-12 \\ \Leftrightarrow x &= 6 \end{aligned}$$

- A. $x = 1$ B. $x = 5$ **C. $x = 6$** D. $x = 14$

3. Simplify: $x^{-6}x^3 = x^{-3} = \frac{1}{x^3}$

- A. x^3 B. $-x^3$ **C. $\frac{1}{x^3}$** D. $-\frac{1}{x^3}$

4. Simplify $\frac{24x^6y^3}{-6x^3y} = -4x^3y^2$

- A. $-4x^2y^3$ **B. $-4x^3y^2$** C. $-4x^3y^3$ D. $-4x^9y^4$

5. Simplify $(a-b)^2 = (a-b)(a-b) = a^2 - ab - ba + b^2 = a^2 - 2ab + b^2$

- A. $a^2 + b^2$ B. $a^2 - b^2$ **C. $a^2 - 2ab + b^2$** D. $a^2 + 2ab - b^2$

6. Simplify: $(5x^2 - 7x + 9) - (-2x^2 - 3x + 2) = 5x^2 - 7x + 9 + 2x^2 + 3x - 2 = 7x^2 - 4x + 7$

7. Simplify. Give your answers using positive exponents only: $(-2x^2y^{-3}w^{-2})^{-3} = (-2)^{-3}x^{-6}y^9w^6$

8. Simplify: $\frac{30x^9 + 8x^7 - 2x^5}{-2x^5} = -15x^4 - 4x^2 + 1$

9. Multiply: $(x-1)(x^2-2x+3) = x^3 - 2x^2 + 3x - x^2 + 2x - 3 = x^3 - 3x^2 + 5x - 3$

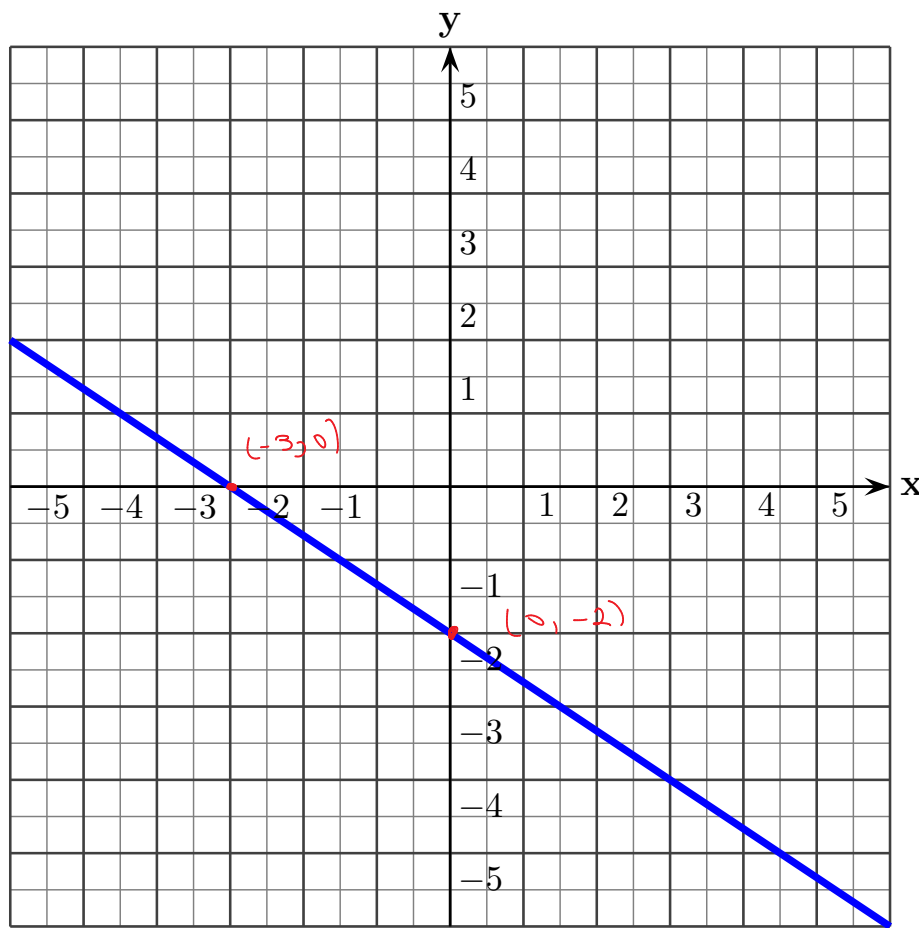
10. Which of the following is a factor of the polynomial $10x^2 - 21x - 10$. $a = -10, b = -21 \Rightarrow -2, 5, 4$

- A. $2x + 5$ B. **$5x + 2$** C. $x - 25$ D. $5x - 2$

11. Which of the following is a factor of the polynomial $6ax - 2a + 5b - 15bx$.

- A. $3x + 1$ B. $5a - 2b$ C. $2a + 5b$ D. **$3x - 1$**

12. What is the slope of the line graphed below?



$$m = \frac{(-2) - (0)}{(0) - (-3)} = \frac{-2}{3} = -\frac{2}{3}$$

- A. $\frac{2}{3}$ **B. $-\frac{2}{3}$** C. $\frac{3}{2}$ D. $-\frac{3}{2}$

13. Find the graph of the solution to the inequality.

$$-5x + 6 < -2x + 3$$

Border value. $-5x + 6 = -2x + 3$

$$\Leftrightarrow -5x + 2x = 3 - 6$$

$$\Leftrightarrow \frac{-3x}{-3} = \frac{-3}{-3}$$

$$\Leftrightarrow x = 1$$

Test point $x=0$

$$6 < 3 \quad \text{False}$$

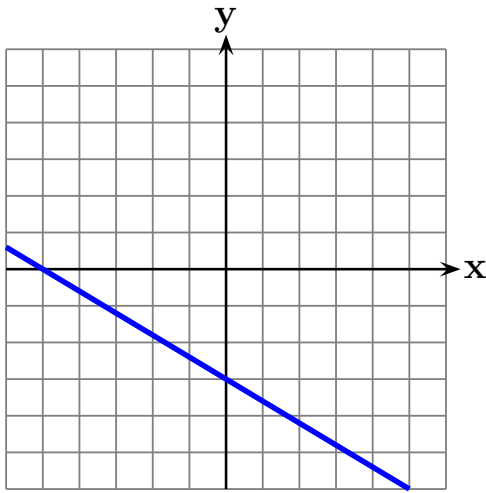
- A)**
- B)
- C)
- D)

14. Which of the following is the graph of the equation?

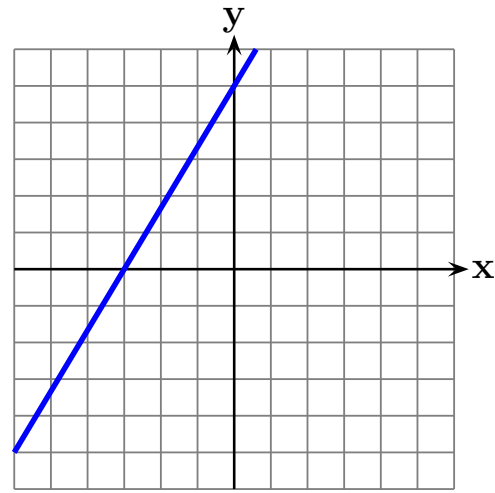
$$5x + 3y = -15$$

$$\begin{array}{r} x \\ 5 \overline{) -15} \\ \underline{-15} \\ 0 \end{array}$$

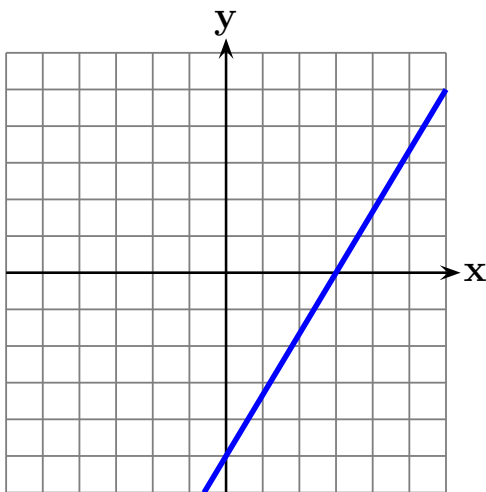
$$\begin{aligned} x=0 &\Rightarrow 3y = -15 \Rightarrow y = -5 \\ y=0 &\Rightarrow 5x = -15 \Rightarrow x = -3 \end{aligned}$$



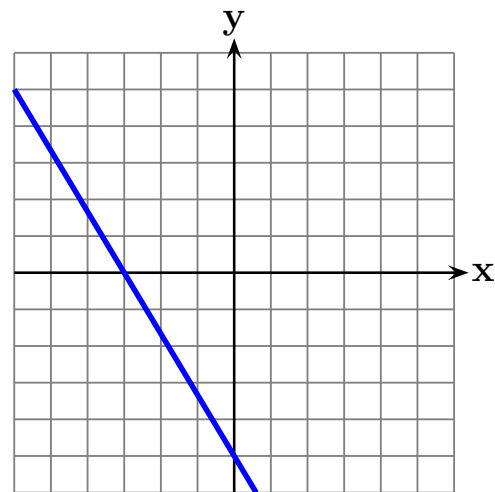
(A)



(B)



(C)



(D)

15. What is the value of the y -coordinate of the solution to the following system of equations?

$$\begin{cases} x + 5y = 17 \\ -2x - 2y = -2 \end{cases}$$

$$\begin{array}{r} 2x + 10y = 34 \\ -2x - 2y = -2 \\ \hline 8y = 32 \end{array}$$

$$\frac{8y}{8} = \frac{32}{8} \Rightarrow y = 4$$

- A. $y = -4$ **B. $y = 4$** C. $y = 16$ D. $y = -16$

16. Solve for x : $18 - 5x = -3(x - 2) \Leftrightarrow 18 - 5x = -3x + 6 \Leftrightarrow 18 - 6 = -3x + 5x$

$$\Leftrightarrow \frac{12}{2} = \frac{2x}{2} \Leftrightarrow 6 = x$$

- A. $x = 10$ **B. $x = 6$** C. $x = -12$ D. $x = 12$

17. Simplify as much as possible:

$$\begin{aligned} (x-2)^2 &= (x-2)(x-2) \\ &= x^2 - 2x - 2x + 4 \\ &= x^2 - 4x + 4 \end{aligned}$$

$$\begin{aligned} (x+2)^2 &= (x+2)(x+2) \\ &= x^2 + 2x + 2x + 4 \\ &= x^2 + 4x + 4 \end{aligned}$$

$$\begin{aligned} &3x(x-2)^2 - 6x(x+2)^2 + 3x(x^2+3x-8) \\ &= 3x(x^2-4x+4) - 6x(x^2+4x+4) + 3x(x^2+3x-8) \\ &= \cancel{3x^3} - 12x^2 + 12x - \cancel{6x^3} - 24x^2 - 24x + \cancel{3x^3} + 9x^2 - 24x = -27x^2 - 36x \end{aligned}$$

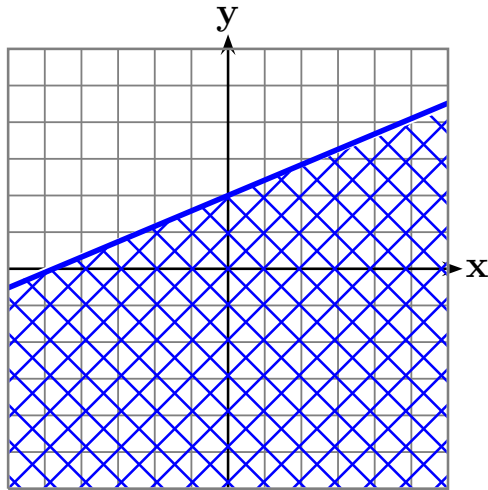
18. Simplify. Give your answers using positive exponents only:

(a) $(2x^{-3}y^2z^{-1})^{-3} = 2^{-3}x^9y^{-6}z^3 = \frac{x^9z^3}{8y^6}$

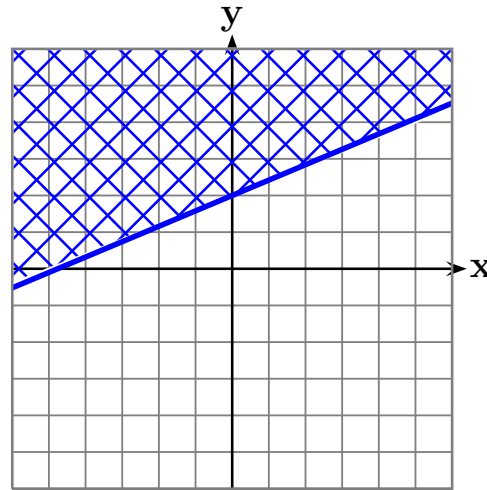
(b) $\frac{9x^{-4}y^3z^{-2}}{6x^{-3}y^5z^3} = \frac{3}{2}x^{-1}y^{-2}z^{-5} = \frac{3}{2xy^2z^5}$

(c) $(3x^2y^3z^{-5})^3 \left(\frac{3x^{-2}y^{-3}z^{-4}}{x^4y^3z^2} \right)^{-2} = 27x^6y^9z^{-15} (3x^{-6}y^{-6}z^8)^{-2} = 27x^6y^9z^{-15} (3^{-2}x^{12}y^{12}z^{-12}) = 3x^{18}y^{15}z^{-3} = \frac{3x^{18}y^{15}}{z^{-3}}$

19. Find the graph of the solution to the inequality: $2x - 5y \geq -10$

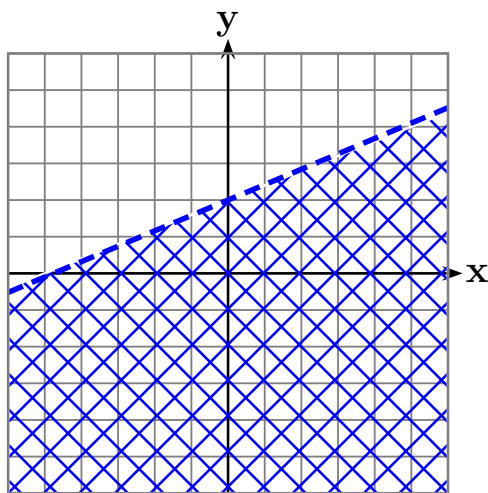


(A)

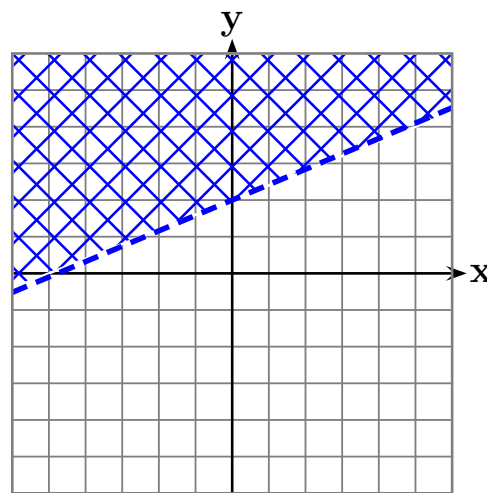


(B)

Test Point (0, 0)
 $0 \geq -10$ TRUE



(C)



(D)

20. Multiply:

(a) $(3x - 5)(5x + 3) = 15x^2 + 9x - 25x - 15 = 15x^2 - 16x - 15$

(b) $(x + 3)(x^2 - 3x + 9) = x^3 - 3x^2 + 9x + 3x^2 - 9x + 27 = x^3 + 27$

(c) $(2x - 3y)(yx^2 - 3xy - xy^2) = 2x^3y - 6x^2y^2 - 2x^2y^2 - 3x^2y^2 + 9xy^3 + 3xy^3 = 2x^3y - 6x^2y^2 - 5x^2y^2 + 9xy^3 + 3xy^3$

(d) $(x - 2)(x^4 + 2x^3 + 4x^2 + 8x + 16) = x^5 + 2x^4 + 4x^3 + 8x^2 + 16x - 2x^4 - 4x^3 - 8x^2 - 16x - 32 = x^5 - 32$

(e) $(x^2 - 3x - 1)(x^2 + 2x + 3) = x^4 + 2x^3 + 3x^2 - 3x^3 - 6x^2 - 9x - x^2 - 2x - 3 = x^4 - x^3 - 4x^2 - 11x - 3$

(f) $(a^2 - b^2)(a^2 + 2ab + b^2) = a^4 + 2a^3b + ab^3 - a^2b^2 - 2ab^3 - b^4$

$$3x - 4y = 12 \Leftrightarrow \frac{3x - 12}{4} = \frac{4y}{4}$$

$$\Leftrightarrow \frac{3x}{4} - 3 = y$$

21. The graph of the line with equation $3x - 4y = 12$ has

- A. Slope $\frac{3}{4}$ and y -intercept $(0, 12)$
- B. Slope $\frac{4}{3}$ and y -intercept $(0, 12)$
- C. Slope $-\frac{3}{4}$ and y -intercept $(0, 3)$
- D.** Slope $\frac{3}{4}$ and y -intercept $(0, -3)$

22. Factor completely:

(a) $64x^2 - 49y^2 = (8x)^2 - (7y)^2 = (8x - 7y)(8x + 7y)$

(b) $x^2 - 15x + 50 = (x - 10)(x - 5)$

(c) $21z^2 - 13z + 2 = 21z^2 - 6z - 7z + 1 = 3z(7z - 2) - 1(7z - 2) = (7z - 2)(3z - 1)$

(d) $5y^2 + 110y + 600 = 5(y^2 + 22y + 120) = 5(y + 10)(y + 12)$

(e) $x^3 + 11x^2 + 30x = x(x^2 + 11x + 30) = x(x + 5)(x + 6)$

(f) $25y^4 - 25y^3 + 6y^2 = y^2(25y^2 - 25y + 6) = y^2(5y - 2)(5y - 3)$

(g) $30x^3 + 21x^2 - 18x = 3x(10x^2 + 7x - 6) = 3x(5x - 6)(2x - 1)$

(h) $27x^3 - 300x = 3x(9x^2 - 100) = 3x(3x - 10)(3x + 10)$

(i) $x^4 - 625 = (x^2)^2 - (25)^2 = (x^2 - 25)(x^2 + 25) = (x - 5)(x + 5)(x^2 + 25)$

(j) $6ax - 10ay + 21bx - 35by = 2a(3x - 5y) + 7b(3x - 5y) = (3x - 5y)(2a + 7b)$

(k) $9xy^2 + 2x - 9xy + 18y^2 - 18y + 4 = x(9y^2 + 2 - 9y) + 2(9y^2 - 9y + 2) = (x + 2)(9y^2 - 9y + 2) = (x + 2)(3y - 2)(3y - 1)$

(l) $x^2y^2 - 25y^2 - yx^2 + 25y - 56x^2 + 1400$

$$x^2(y^2 - y - 56) - 25(y^2 - y - 56) = (x^2 - 25)(y^2 - y - 56)$$

$$= (x - 5)(x + 5)(y - 8)(y + 7)$$

Product 42 } -6, -7
Sum -13

Product 150 } -10, -15
Sum -25

$25y^2 - 10y - 15y + 6$
 $= 5y(5y - 2) - 3(5y - 2)$
 $= (5y - 2)(5y - 3)$

Product -60 } 12, -5
Sum 7
 $10x^2 + 12x - 5x - 6$
 $= 2x(5x + 6) - 1(5x + 6)$
 $= (5x + 6)(2x - 1)$

Product 18 } -6
Sum -9 } -3

$9y^2 - 6y - 3y + 2$
 $= 3y(3y - 2) - 1(3y - 2)$
 $= (3y - 2)(3y - 1)$