Review for the third exam

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Disclaimer The following is a set of questions to help you review what we have covered in class, after the second exam. The third exam will also contain some topics that were covered in the first two exams, the reviews for these parts are here: Review for the first exam, and Review for the second exam.

If you know how to answer these questions then you should do well in the exam. However there is no guarantee that the questions in the actual exam will be perceived to be similar to these questions.

1. Consider the following equation:

$$6x - 5y = 30$$

(a) Complete the following table of solutions of this equation:

$x=0 \Rightarrow -5y=30$
$\Rightarrow y=\frac{30}{-5}$
$\Rightarrow y=-6$
$y=0 \Rightarrow 6x=30$
$\Rightarrow x=5$

$x=-1 \Rightarrow 6(-1)-5y=30$
$\Rightarrow -6-5y=30$
$\Rightarrow -5y=36$
$\Rightarrow y=-\frac{36}{5}$

$x=1 \Rightarrow 6(1)-5y=30$
$\Rightarrow 6-5y=30$
$\Rightarrow -5y=24$
$\Rightarrow y=-\frac{24}{5}$

$y=1 \Rightarrow 6x-5(1)=30$
$\Rightarrow 6x-5=30$
$\Rightarrow 6x=35$
$\Rightarrow x=\frac{35}{6}$

x	y
0	-6
5	0
1	$-\frac{24}{5}$
-1	$-\frac{36}{5}$
$\frac{35}{6}$	1

(b) The second coordinate of a solution to this equation equals twice the first coordinate. Find that solution.

Answer. If (x, y) are the coordinates of this solution, we have $y = 2x$. So plugging $2x$ instead of y in the equation we get:

$$\begin{aligned} 6x - 5(2x) = 30 &\iff 6x - 10x = 30 \\ &\iff -4x = 30 \\ &\iff x = -\frac{30}{4} \\ &\iff x = -\frac{15}{2} \end{aligned}$$

So the first coordinate is $-\frac{15}{2}$. The second coordinate is twice that, or $y = -15$. So the solution is $(-\frac{15}{2}, -15)$. □

(c) There are two solutions to this equation with coordinates that are consecutive integers. Find those solutions.

Answer. The second cordinate will be either one more or one less than the first. In the first case we have that $y = x + 1$. Then plugging $x + 1$ instead of y in the equation gives:

$$\begin{aligned} 6x - 5(x+1) = 30 &\iff 6x - 5x - 5 = 30 \\ &\iff x - 5 = 30 \\ &\iff x = 35 \end{aligned}$$

So one of the solutions is $(35, 36)$.

In the second case we have $y = x - 1$. Then plugging $x - 1$ instead of y in the equation gives:

$$\begin{aligned} 6x - 5(x-1) = 30 &\iff 6x - 5x + 5 = 30 \\ &\iff x + 5 = 30 \\ &\iff x = 25 \end{aligned}$$

So the second solution is $(25, 24)$. □

2. Find five solutions for each of the equations:
   (a) $-3x + 2y = 6$
   (b) $2x - 5y = -3$
   (c) $y = \dfrac{2x}{3} - 7$

Answer. This question has many correct answers. One possible answer for each equation is shown below: □

a)

x	y
0	3
-2	0
1	$\frac{9}{2}$
4	9
2	6

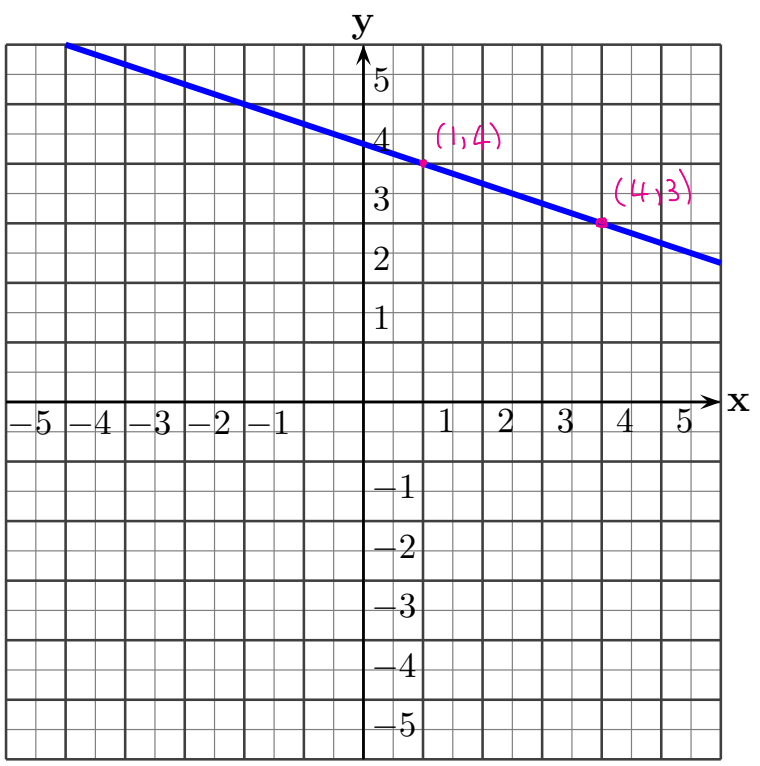
b)

x	y
0	$\frac{3}{5}$
$-\frac{3}{2}$	0
1	-1
-1	$\frac{1}{5}$
2	$-\frac{1}{5}$

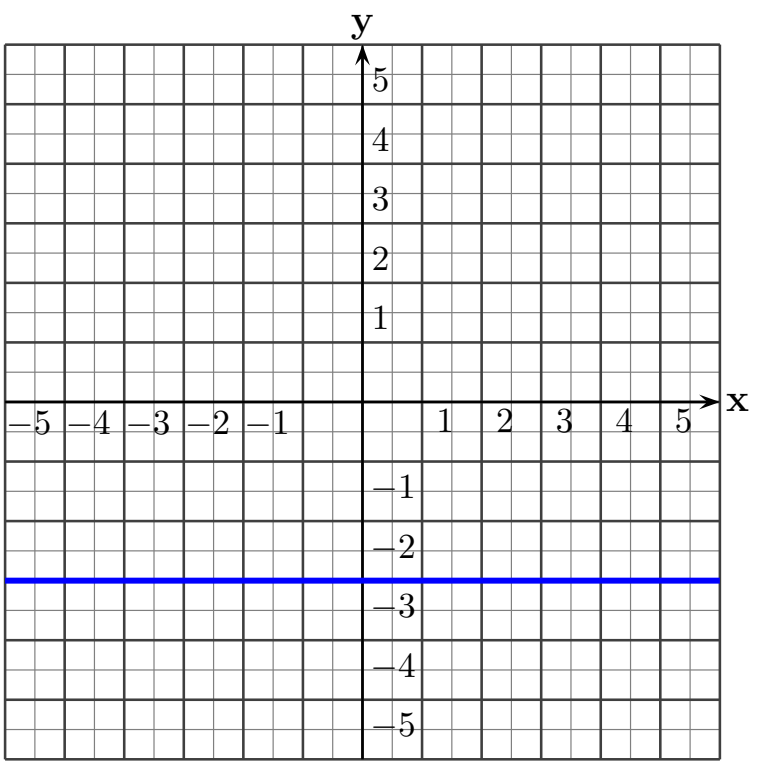
c)

x	y
0	-7
3	-5
-3	-9
6	-3
-6	-11

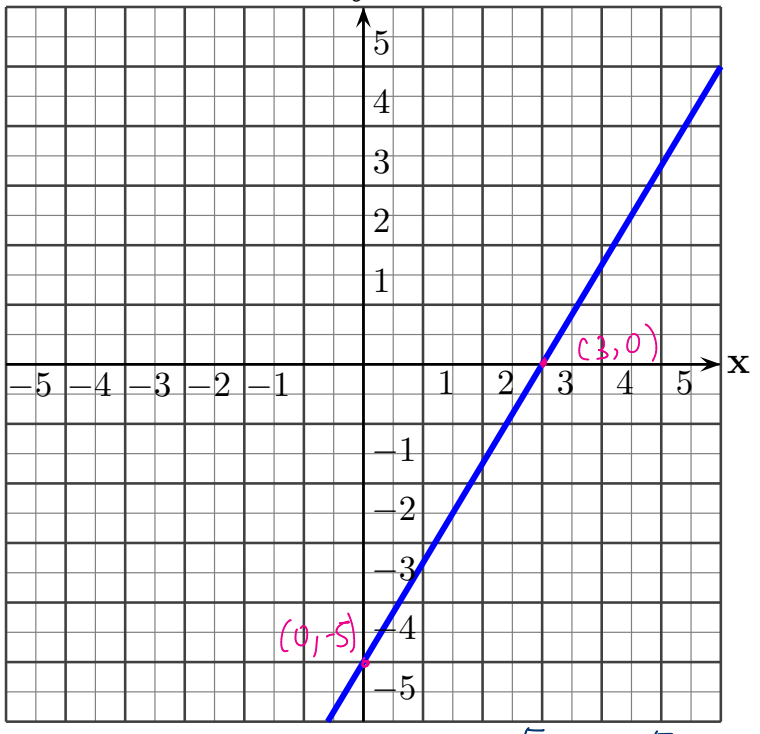
3. Find the slope for each of the lines whose graph is shown below:



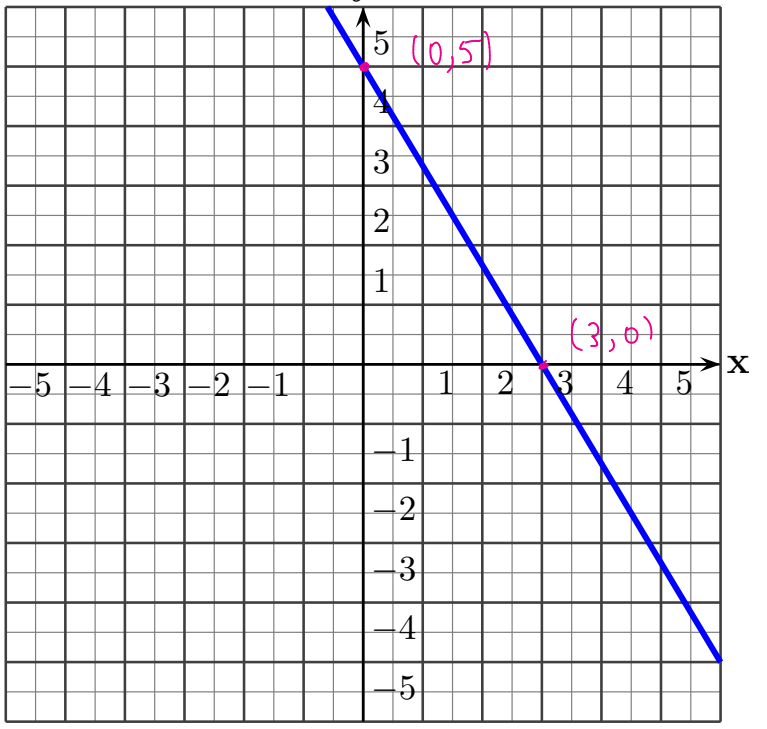
A $m = \frac{3-4}{4-1} = -\frac{1}{3}$



B $m = 0$



C $m = \frac{-5-0}{0-3} = \frac{5}{3}$



D $m = \frac{0-5}{3-0} = -\frac{5}{3}$

4. Find an equation for each of the lines of the previous question.

A) $y = -\frac{x}{3} + b$

Plug $(1,4)$ $\quad 4 = -\frac{1}{3} + b$

$\Rightarrow b = 4 + \frac{1}{3}$

$\Rightarrow b = \frac{13}{3}$

$y = -\frac{x}{3} + \frac{13}{3}$

B) $y = -3$

C) $y = \frac{5}{3}x - 5$

D) $y = -\frac{5}{3}x + 5$

5. Graph each of the following equations:

(a) $x + y = 2$

(b) $3x + 2y = 6$

(c) $-2x + 5y = -10$

(d) $y = -\frac{3}{4}x + 5$

(e) $x = 3$

(f) $y = -3$

a)

x	y
0	2
2	0

b)

x	y
0	3
2	0

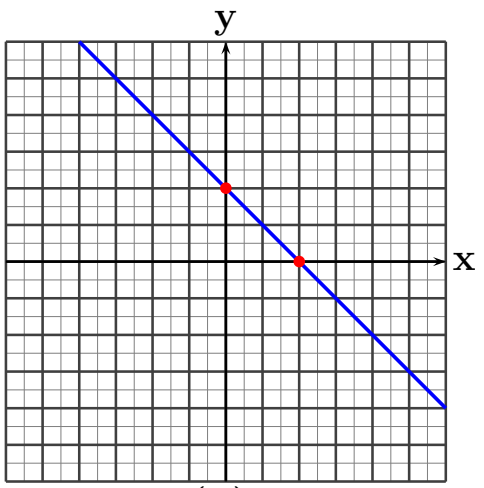
c)

x	y
0	-2
5	0

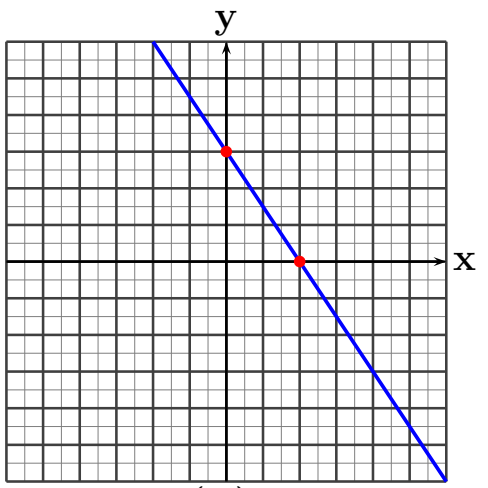
d)

x	y
0	5
4	2

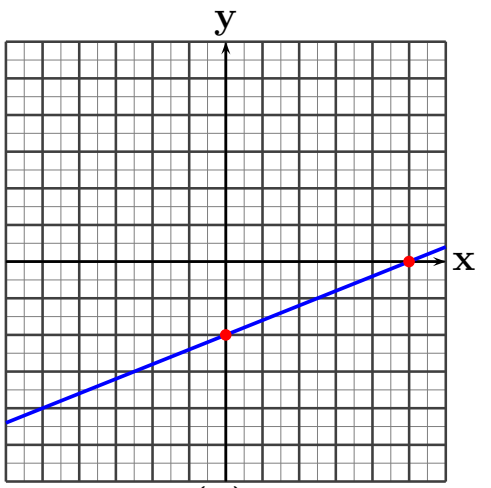
For each of the first four lines you have to find two solutions to the equation. One possible choice is shown: for the first three lines we find the two intercepts. For the fourth we choose $x = 0$ and $x = 4$. The fifth is a vertical line and the sixth horizontal.



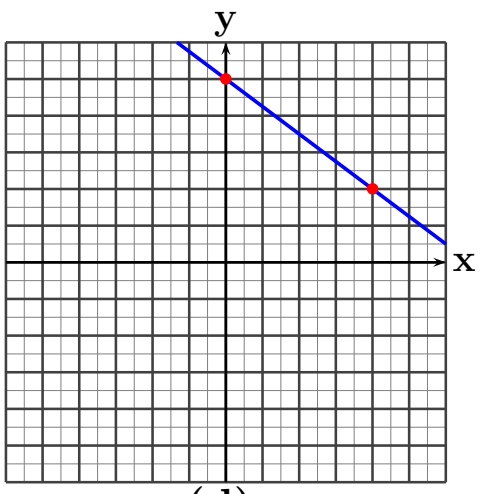
(a)



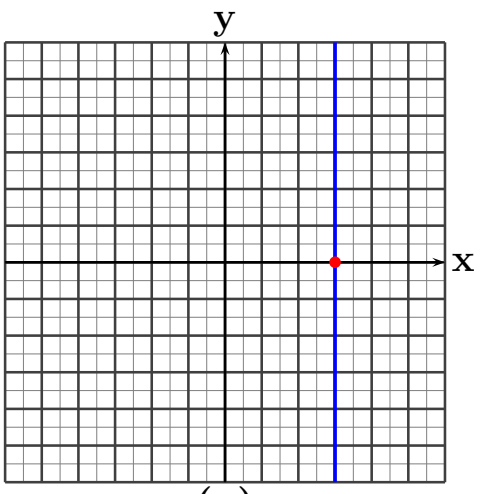
(b)



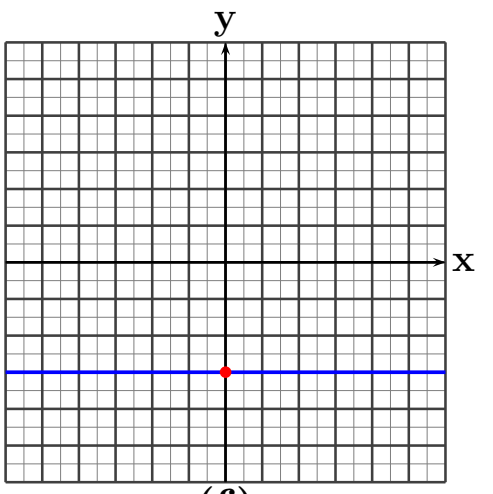
(c)



(d)



(e)



(f)

f) $\begin{array}{c|c} x & y \\ \hline 0 & 6 \\ -10 & 0 \end{array}$ $m = \frac{(0)-(6)}{(-10)-(0)} = -\frac{6}{10} = -\frac{3}{5}$

d) $\begin{array}{c|c} x & y \\ \hline 0 & -6 \\ -14 & 0 \end{array}$ $m = \frac{(0)-(-6)}{(-14)-(0)} = -\frac{6}{14} = -\frac{3}{7}$

e) $\begin{array}{c|c} x & y \\ \hline 0 & -\frac{3}{2} \\ 3 & 0 \end{array}$ $m = \frac{(0)-(-\frac{3}{2})}{(3)-(0)} = \frac{\frac{3}{2}}{3} = \frac{3}{2}\cdot\frac{1}{3} = \frac{1}{2}$

6. Find the slope and the x– and y–intercepts of the line with equation

   (a) $2x - 5y = 20$ a) $\begin{array}{c|c} x & y \\ \hline 0 & -4 \\ 10 & 0 \end{array}$ $m = \frac{(0)-(-4)}{10-0} = \frac{4}{10} = \frac{2}{5}$

   (b) $3x - 2y = 7$ b) $\begin{array}{c|c} x & y \\ \hline 0 & -\frac{7}{2} \\ \frac{7}{3} & 0 \end{array}$ $m = \frac{(0)-(-\frac{7}{2})}{(\frac{7}{3})-(0)} = \frac{\frac{7}{2}}{\frac{7}{3}} = \frac{7}{2}\cdot\frac{3}{7} = \frac{3}{2}$

   (c) $-3x + 4y = 12$ c) $\begin{array}{c|c} x & y \\ \hline 0 & 3 \\ -4 & 0 \end{array}$ $m = \frac{(0)-(3)}{(-4)-(0)} = \frac{3}{4}$

   (d) $3x + 7y = -42$

   (e) $-x + 2y = -3$

   (f) $-3x + 5y = 30$

7. A line has slope $\frac{2}{3}$ and passes through the point $(0, -4)$. Find it's equation. $y = \frac{2x}{3} - 4$

8. A line has slope -3 and passes through the point $(1, 7)$. Find its equation. $y = -3x + 10$

9. A line passes through the points with coordinates $(2, -3)$ and $(-1, 3)$. Find its equation. $y = -2x + 1$

10. A line passes through the points with coordinates $(0, 2)$ and $(-2, 3)$. Find its equation. $y = -\frac{x}{2} + 2$

11. A vertical line passes through the point $(-1, 3)$. Find it's equation. $x = -1$

12. A horizontal line passes through the point $(-6, -7)$. Find it's equation. $y = -7$

13. A line is parallel to the line with equation $y = -3x + 6$ and meets the y–axis at the same point as the line with equation $y = 5x - 7$. Find an equation for this line. $y = -3x - 7$

14. Find the slope and the y intercept of the graph of the equation $3x + 4y = 8$ $\Leftrightarrow$ $4y = -3x + 8$ $\Leftrightarrow$ $y = \frac{-3x+8}{4}$ $\Leftrightarrow$ $y = -\frac{3}{4}x + 2$

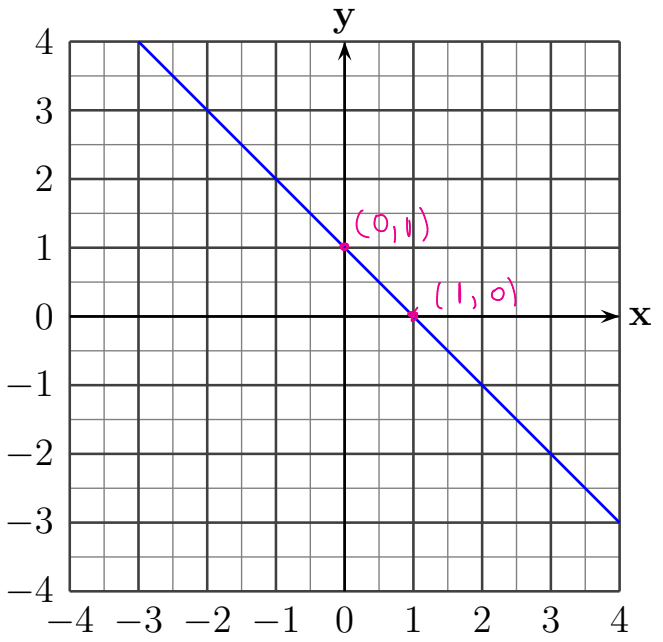
   A. slope$= -\frac{3}{4}$ and y-intercept $(0, 2)$

   B. slope$= \frac{4}{3}$ and y-intercept $(0, 8)$

   C. slope$= \frac{3}{4}$ and y-intercept $(0, 2)$

   D. slope$= -\frac{3}{3}$ and y-intercept $(0, 8)$

15. Choose the correct equation for the line whose graph is shown below:



Check intercepts.

A. $x - y = 1$

x	y
0	−1
1	0

B. $x + y = 1$

x	y
0	1
1	0

C. $x + y = -1$

x	y
0	−1
−1	0

D. $x - y = -1$

x	y
0	1
−1	0

16. Which of the following is the graph of the equation?

$3x - 2y = 6$

Check intercepts

x	y
0	−3
2	0

(A)

(B)

(C)

(D)

17. Graph the solution set of each of the following inequalities:

(a) $3x - 2y \geq 6$ $0 \geq 6$ (F)

(b) $y - \dfrac{3x}{2} < 4$ $0 < 4$ (T)

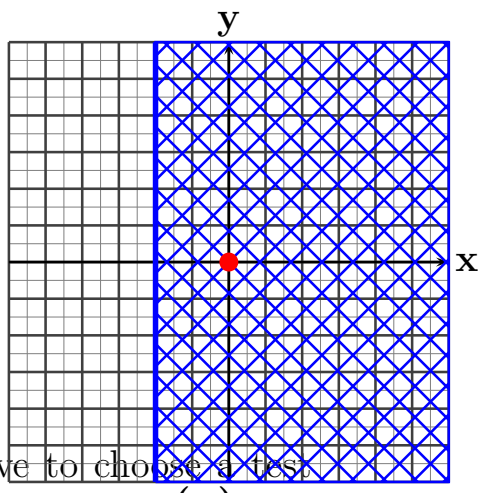
(c) $y \leq 2x - 1$ $0 \leq -1$ (F)

(d) $y < 1$ $0 < 1$ (T)

(e) $4x - 3y \leq 12$ $0 \leq 12$ (T)

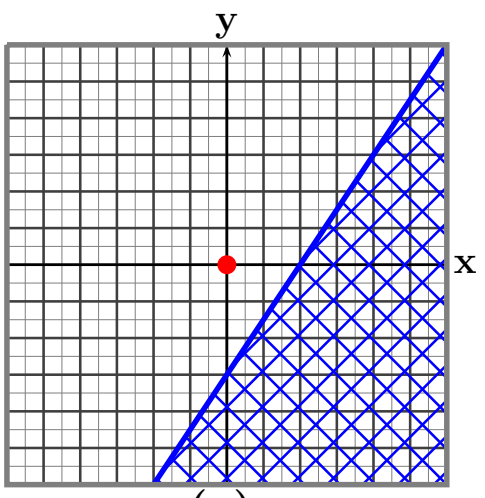
(f) $5x > -2y + 10$ $0 > 10$ (F)

(g) $x \geq -2$ $0 \geq -2$ (T)

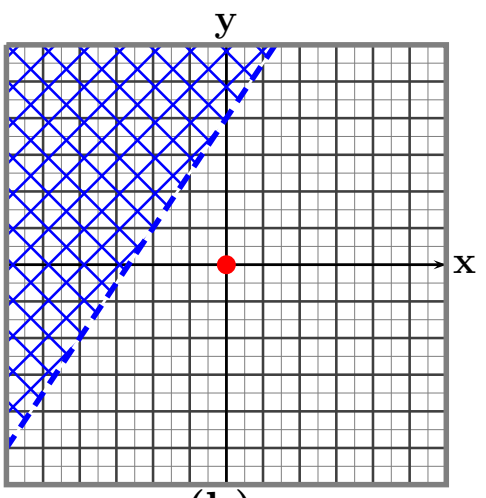


(g)

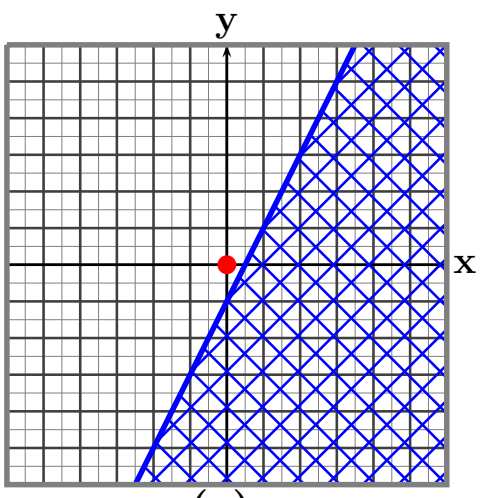
Answer. We first graph the border line for each case. Then we have to choose a test point outside the border line to decide which side is the side of the solutions. In each case we chose the origin $(0, 0)$ as a Test Point, since it doesn't lie in any of the border lines. □



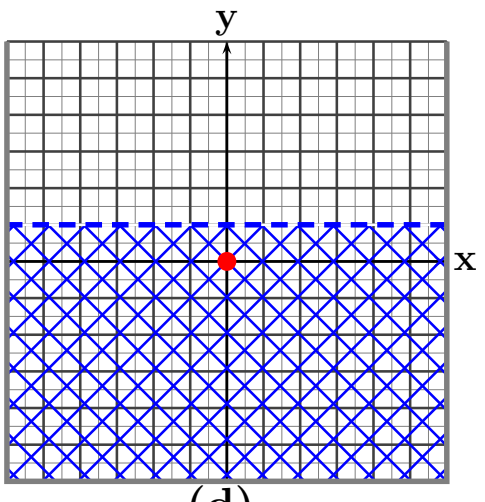
(a)



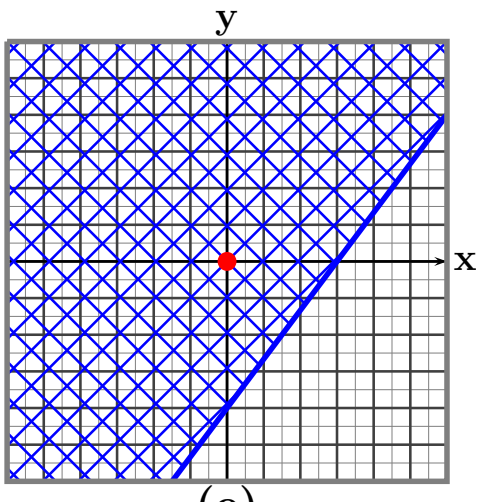
(b)



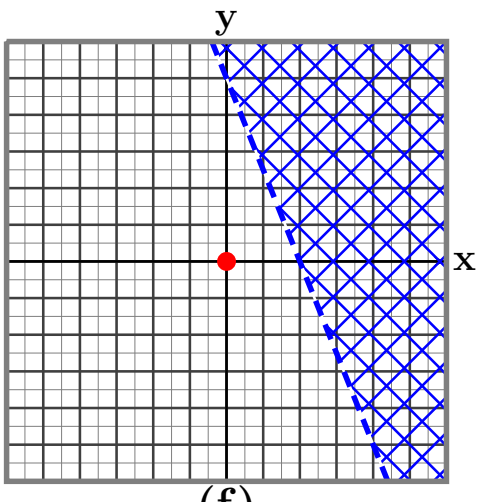
(c)



(d)



(e)



(f)

18. Match the inequalities with their graphs:

(a) $2x + 3y \geq -2$ $0 \geq -2$ (T) (B)

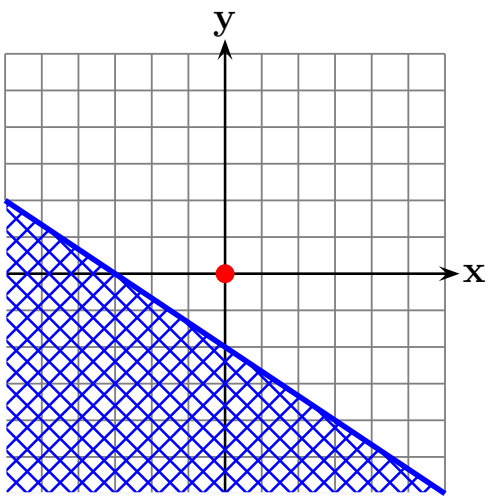
(b) $2x + 3y < -2$ $0 < -2$ (F) (C)

(c) $2x + 3y \leq -2$ $0 \leq -2$ (F) (A)

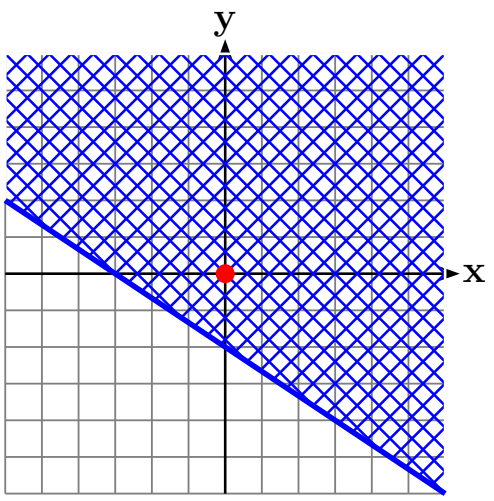
(d) $2x + 3y > -2$ $0 > -2$ (T) (D)

Answer. (b) and (d) are strict inequalities, so the border line is not included in the solution set and so it has to be one of the "broken" lines, so they are matched with (C) and (D). Also (a) and (c) are composite inequalities so their solution set includes the border line, and therefore they have to be matched with the graphs with solid border, (A) and (B).

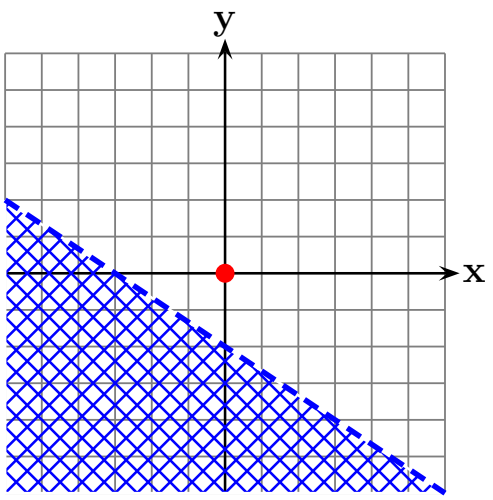
To determine further which inequality is matched with which graph we use the Test Point $(0, 0)$. □



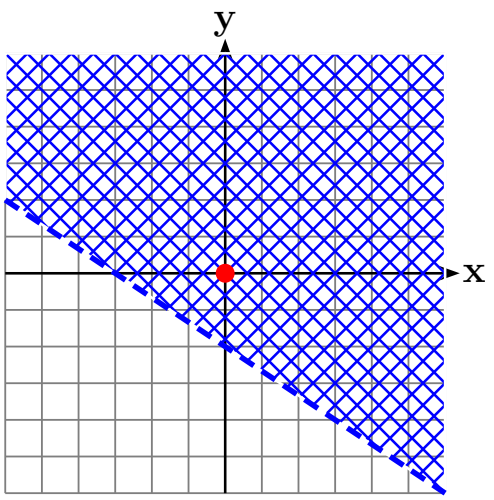
(A)



(B)



(C)



(D)

19. What is the value of the y-coordinate of the solution to the following system of equations?

$$\begin{cases} x + 3y = 2 \\ -3x - 8y = 4 \end{cases}$$

$3x + 9y = 6$
$-3x - 8y = 4$
$y = 10$

A. $y = -2$ B. $y = 10$ C. $y = 6$ D. $y = -10$

20. What is the value of the x-coordinate of the solution to the following system of equations?

$$\begin{cases} 2x + y = 3 \\ -5x - 2y = 4 \end{cases}$$

$4x + 2y = 6$
$-5x - 2y = 4$
$-x = 10 \Rightarrow x = -10$

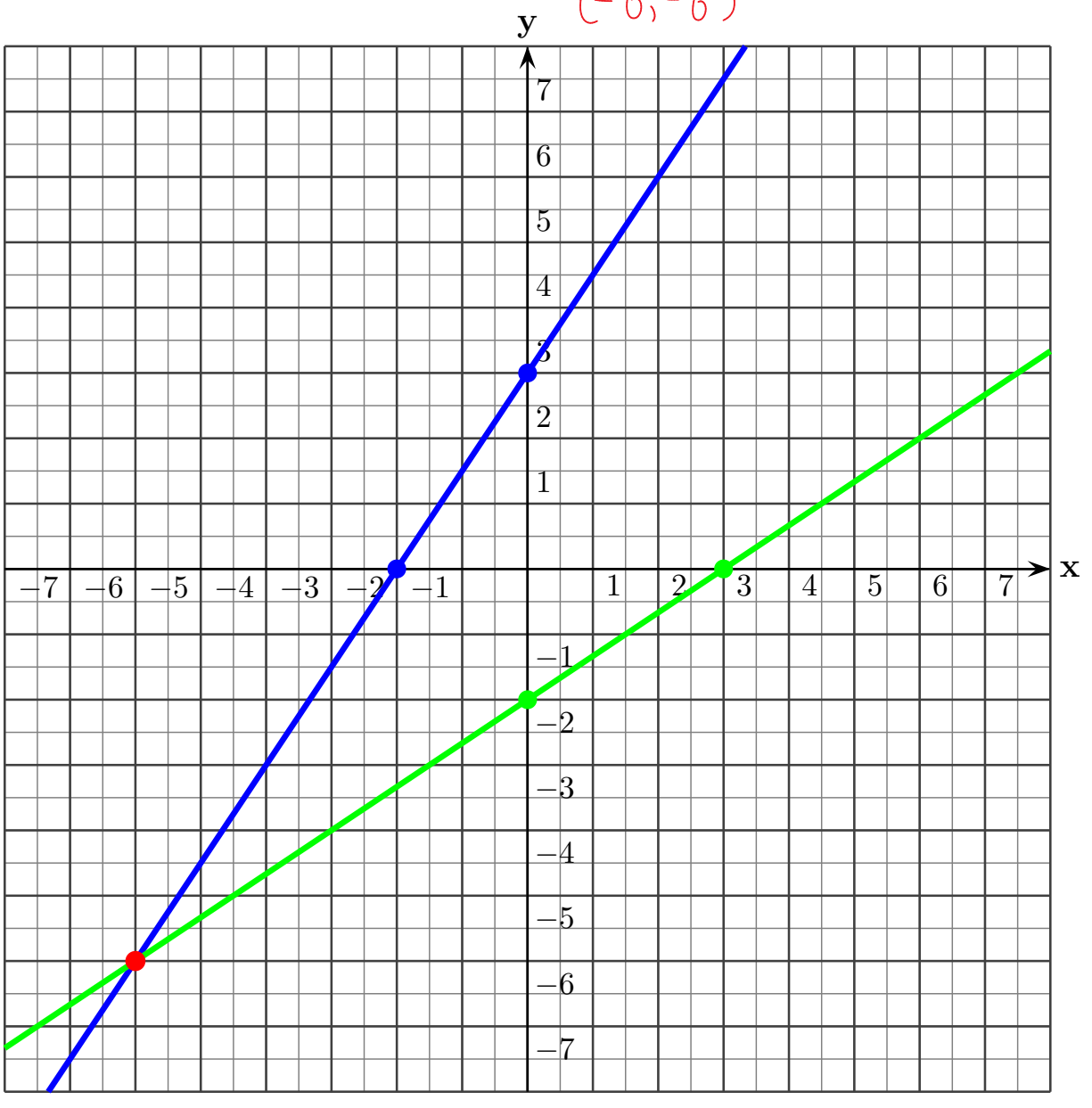
A. $x = 2$ B. $x = -10$ C. $x = 10$ D. $x = -7$

21. Solve the system: $\begin{cases} 2x - 3y = -10 \\ 3x + 2y = -2 \end{cases}$

$-6x + 9y = 30$
$6x + 4y = -4$
$13y = 26 \Rightarrow y = 2$

$4x - 6y = -20$
$9x + 6y = -6$
$13x = -26$
$\Rightarrow x = -2$

22. Graph the lines with equations $-3x + 2y = 6$ and $2x - 3y = 6$ in the following grid. At what point do the two lines meet? $(-6, -6)$



a) $-6x+4y=12$, $6x-9y=18$

$-5y=30 \Rightarrow \boxed{y=-6}$

$-9x+6y=18$, $4x-6y=12$

$-5x=30 \Rightarrow \boxed{x=-6}$

b) $6x+6y=9$, $-6x+6y=-8$

$12y=18 \Rightarrow y=\frac{18}{12} \Rightarrow \boxed{y=\frac{3}{4}}$

$4x+4y=6$, $6x-4y=8$

$10x=14 \Rightarrow x=\frac{14}{10} \Rightarrow \boxed{x=\frac{7}{5}}$

23. Solve each of the following systems:

(a) $\begin{cases} -3x+2y=6 & 3 \\ 2x-3y=6 & 2 \end{cases}$ (2, 3)

(b) $\begin{cases} 2x+2y=3 & 2 \\ 3x-2y=4 & 2 \end{cases}$ (3, -2)

(c) $\begin{cases} -2x+5y=3 & 2 \\ -5x+2y=-3 & -5 \end{cases}$ (5, 2)

(d) $\begin{cases} 5x+7y=29 \\ -10x-7y=-51 \end{cases}$ (2)

c) $-10x+25y=15$, $10x-4y=6$

$21y=21 \Rightarrow \boxed{y=1}$

$-4x+10y=6$, $25x-10y=15$

$21x=21 \Rightarrow \boxed{x=1}$

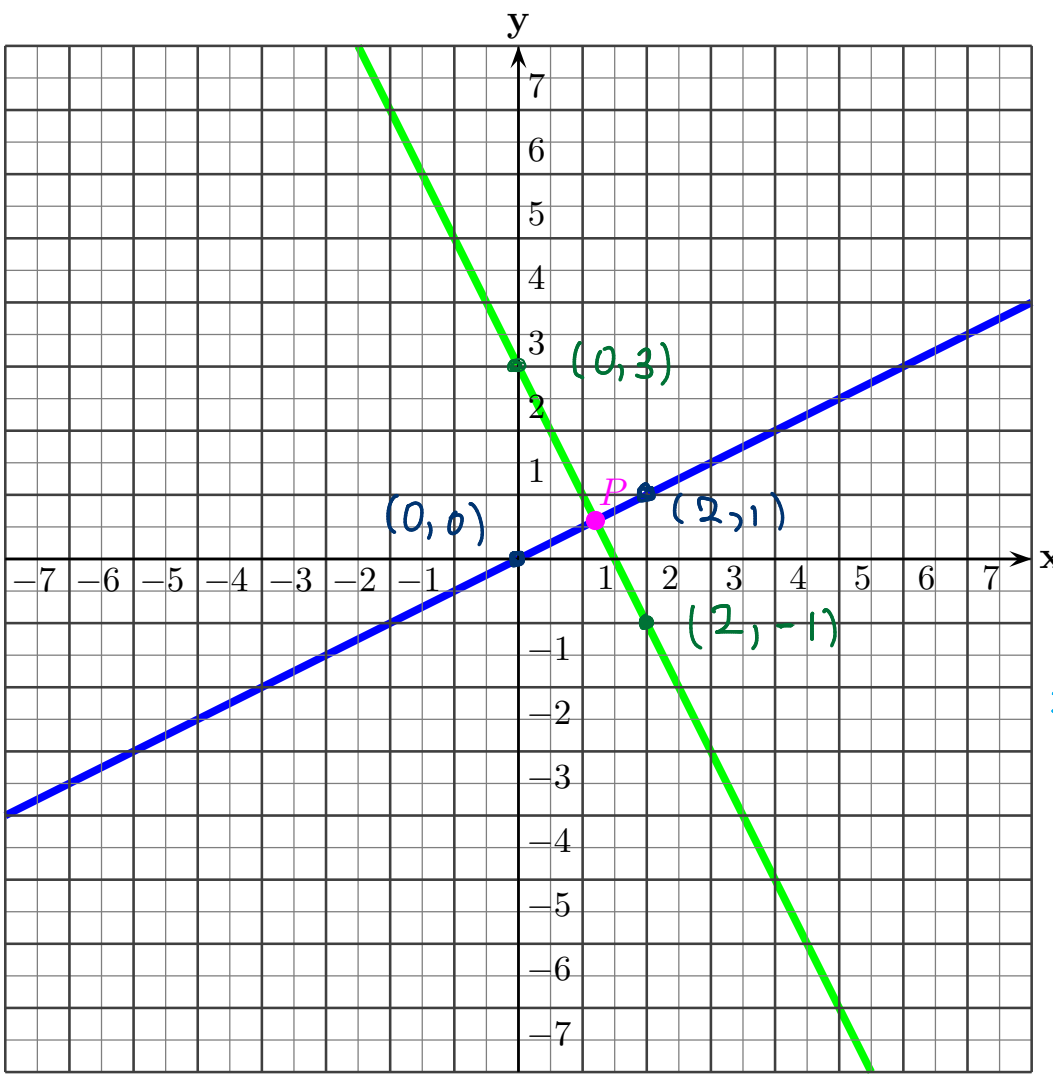
d) $10x+14y=58$, $-10x-7y=-51$

$7y=7 \Rightarrow \boxed{y=1}$

$5x+7y=29$, $-10x-7y=-51$

$-5x=-21 \Rightarrow \boxed{x=\frac{21}{5}}$

24. Consider the two lines whose graphs are shown in the same grid bellow:



$m=\frac{(-1)-(3)}{(2)-(0)}=\frac{-4}{2}=-2$

$y=-2x+3 \Leftrightarrow \boxed{2x+y=3}$

$m=\frac{(1)-(0)}{(2)-(0)}=\frac{1}{2}$

$y=\frac{x}{2} \Leftrightarrow 2y=x$

$\Leftrightarrow \boxed{-x+2y=0}$

$2\begin{cases} 2x+y=3 \\ -x+2y=0 \end{cases}$ -2

$2x+y=3$, $-2x+4y=0$

$5y=3 \Rightarrow \boxed{y=\frac{3}{5}}$

$-4x-2y=-6$, $-x+2y=0$

$-5x=-6 \Rightarrow \boxed{x=\frac{6}{5}}$

(a) Write an equation in standard form for each of the lines

(b) Solve the system of those two equations $x=\frac{6}{5}$, $y=\frac{3}{5}$

(c) What are the coordinates of the point P? $\left(\frac{6}{5}, \frac{3}{5}\right)$

(d) What is the angle between the two lines? Because the slopes of the two lines are negative reciprocal the two lines are perpendicular. So the angle is 90°