

Publication List

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Publications:

- [1] 2013, *Non-commutative Hilbert modular symbol*, arXiv:1308.4991 [math.NT], 50 pages, to appear in **Algebra and Number Theory**.

I construct an analogue of Manin's non-commutative modular symbol for Hilbert modular groups of the type $SL_2(\mathcal{O}_K)$, where \mathcal{O}_K is the ring of integers in a real quadratic field K . In order to do that, I define a higher dimensional analogue of iterated path integrals, which I call iterated integrals over membranes. Using the latter, I define non-commutative Hilbert modular symbol. Manin relates his non-commutative modular symbol to multiple zeta values in two ways - by infinite sum and by intreated path integrals. Similarly, I relate the non-commutative Hilbert modular symbol to multiple Dedekind zeta values (see preprint [4]) - by infinite sums and by iterated integrals over membranes.

- [2] 2014, *Multiple Dedekind Zeta Functions*, **Crelle's Journal, (Journal fuer die reine und angewandte Mathematik)**, 43 pages, to appear.

In this paper, I define a multiple Dedekind zeta values via higher dimensional generalization of iterated path integrals, which I call iterated integrals over membranes. I give many particular formulas for such values over the Gaussian integers. Then I interpolate the multiple Dedekind zeta values into multiple Dedekind zeta functions. I prove analytic continuation and compute multiple residues at $(1, 1; 1, 1)$ for a double Dedekind zeta value over a quadratic field. This multiple residue can be written as a product of two logarithm of an elements of a quadratic field, each to them resembling a regulator, and a discriminant.

- [3] 2014, *Reciprocity Laws on Algebraic Surfaces via Iterated Integrals*, 30 pages, **Journal of K-theory**, to appear.

In this paper, I prove classical and new reciprocity laws on algebraic surfaces, using iterated integrals over membranes. I use the fundamental, instead of Galois group. In order to capture non-abelian parts of the fundamental group, I use iterated integrals. We prove reciprocity laws both for the Parshin symbol and for the new 4-function local symbol defined by me, in essentially the same way - via a new approach using intreated integrals on membranes. I also construct a refinement of the Parshin symbol and use it to prove the Parshin reciprocity laws and the 4-function local symbol.

In the Appendix, we provide an alternative proof for the reciprocity of a new 4-function local symbols on a surface, using Milnor K -groups.

- [4] 2014, *Cohomology of $GL_4(\mathbb{Z})$ with non-trivial coefficients*, **Mathematical Research Letters**, to appear.

In this paper I compute the cohomology groups of the arithmetic group $GL_4(\mathbb{Z})$ with coefficients in symmetric powers of the standard representation twisted by the determinant representation. Such results are needed for Goncharov's approach to examine the dimensions of certain spaces of motivic multiple zeta values. I have used Borel-Serre compactification, Kostant's theorem and many spectral sequences.

- [5] 2013, (with Anton Deitmar), *Iterated integrals and higher order invariants*, **Canad. J. Math.** 65, 544-552, <http://dx.doi.org/10.4153/CJM-2012-020-8>.

In this paper, we state a conjecture relating iterated integral on a manifold, iterated integrals on its universal cover and higher order invariant. We prove this conjecture in some cases.

- [6] 2011, *Non-abelian reciprocity laws on a Riemann surface*, **Int. Math. Res. Notices**, 2011, no 11, 2469-2495.

Usually, reciprocity laws are related to a Galois group. In Grothendieck's philosophy a Galois group and a fundamental group are very similar objects. (There is an intermediate object - the étale fundamental group.) In this paper, I use the fundamental group of a Riemann surface, in order to prove classical and new reciprocity laws. I prove a reciprocity law for a generating series of iterated integrals on a Riemann surfaces. In degrees 1 and 2 this captures several known reciprocity laws: sum of the residues, Weil reciprocity, Riemann relations. In degree 3, I prove a new reciprocity law.

- [7] 2005, *Euler characteristics of arithmetic groups*, **Math. Res. Lett.**, Vol. 12, issue 3, 275-291.

This paper is an abbreviated version of my thesis. Here, I present a computationally effective method for finding the homological Euler characteristic of a large class of arithmetic groups, essentially, $GL_n(R)$ and $SL_n(R)$, where R is the ring of integers in a number field. I prove vanishing theorems. The results were needed for Goncharov's approach to examining the dimensions of certain spaces of motivic multiple zeta values.

- [8] 1993, (in high-school) *Cubes in an integer lattice*, **Mathematics and Informatics**, vol. 3 no 3, 85-89.

In this paper, I parametrize all cubes with integer coordinates of their vertices, using arithmetic over the Gaussian integers. It has been cited.

Submitted:

- [1] 2013, *Multiple Zeta Values and Ideles*, arXiv:math/0611849 [math.NT], 10 pages.

In this paper I give two idelic representation of the multiple zeta values - one using iterated integrals over the the finite ideles and the other using iterated integrals over idele class group. I obtain two types of shuffle relations of multiple zeta values via two idelic representations.

- [2] 2014, *Double Shuffle Relations for Multiple Dedekind Zeta Values*, arXiv:1311.4019 [math.NT], 30 pages.

A product of two multiple zeta values can be expressed as a sum of such in two ways - one using the infinite sum representation and another using the iterated integrals representation. This properties are called double shuffle relations. It was expected that multiple Dedekind zeta values also have double shuffle relations. In this paper, I present double shuffle relations for multiple Dedekind zeta values associated to imaginary quadratic fields. I also present new linear relations among multiple Dedekind zeta values.

Preprints:

- [1] 2013, (with Zhenbin Luo), *On the Conou-Carrère Symbol for Surfaces*, arXiv:1310.7065 [math.AG], 26 pages.

We use a new method of iterated integrals in order to construct the Contou-Carrere symbol. We find one more new ingredient in the formula for the Contou-Carrere symbol compared to the existing ones.

- [2] 2012, *Parallel Transport on Higher Loop Spaces* arXiv:1206.5784 [math.AT], 17 pages.

I construct a parallel transport on higher loop spaces of a manifold in terms of a higher dimensional generalization of iterated path integrals. However, I was informed that such results were obtained earlier in arXiv:1106.1668 [math.DG].

- [3] 2006, *Non-commutative two-dimensional modular symbol*, arXiv:math/0611955v1 [math.NT], 27 pages. (substantially developed and submitted as two consecutive papers: [2] from Publications and [1] from the Submitted.)

In this paper, I have defined a generalization to Manin's non-commutative modular symbol. It was the first paper, where iterated integrals over membranes were defined. It was also the first paper where I constructed a non-commutative 2-cocycle geometrically via generating series of iterated integrals. I also gave a definition of multiple Dedekind zeta functions for some number fields.

In Preparation:

- [1] Multiple Dedekind zeta values via ideles.
- [2] A book on Iterated integrals in Arithmetic Algebraic Geometry.