COMPUTATION OF VOLUMES OF SOLIDS OF REVOLUTION

<table>
<tr><th rowspan="2">Revolution about a line parallel to</th><th colspan="2">Method</th></tr>
<tr><th>Washers</th><th>Shells</th></tr>
<tr><td>The x-axis</td><td>$V = \pi \int_{x_1}^{x_2} \left[R_{ext}^2(x) - R_{\text{int}}^2(x) \right] dx$</td><td>$V = 2\pi \int_{y_1}^{y_2} (y - y_0)\left[R_{ext}(y) - R_{\text{int}}(y) \right] dy$</td></tr>
<tr><td>The y-axis</td><td>$V = \pi \int_{y_1}^{y_2} \left[R_{ext}^2(y) - R_{\text{int}}^2(y) \right] dy$</td><td>$V = 2\pi \int_{x_1}^{x_2} (x - x_0)\left[R_{ext}(x) - R_{\text{int}}(x) \right] dx$</td></tr>
<tr><td rowspan="2">Notation</td><td colspan="2">$R_{ext}(x)$, $R_{\text{int}}(x)$, $R_{ext}(y)$, $R_{\text{int}}(y)$ denote external or internal radii of revolution, that is, distances of the point revolved from the axis of revolution.</td></tr>
<tr><td colspan="2">x_0 and y_0 denote, respectively, the distance of a variable point of integration (x or y) to the axis of revolution. If we rotate about the x-axis, $y_0 = 0$, if we rotate about the y-axis, $x_0 = 0$.</td></tr>
<tr><td rowspan="2">Remarks</td><td colspan="2">1. The limits must be consistent with the variable of integration
2. We always integrate along</td></tr>
<tr><td>The axis of revolution if the washer technique is used</td><td>The perpendicular axis if the shell technique is used</td></tr>
<tr><td>Example</td><td colspan="2">Let R be the region between $y = 1 + x$, $y = 1/x$, $x = 1$, $x = 2$. Find the volume of the solid of revolution, if we revolve R about</td></tr>
<tr><td>the line $y = -2$, thus $y_0 = -2$</td><td>$V = \pi \int_1^2 \left[(x+1+2)^2 - (\frac{1}{x} + 2)^2 \right] dx =$
$= \left(15\tfrac{5}{6} - 4\ln 2\right)\pi$</td><td>$V = 2\pi \left\{ \int_{1/2}^{1} (y+2)\left[2 - \tfrac{1}{y} \right] dy + \int_1^2 (y+2)[1] dy + \int_2^3 (y+2)[2 - (y-1)] dy \right\} =$
$= \left(15\tfrac{5}{6} - 4\ln 2\right)\pi$</td></tr>
<tr><td>the line $x = -2$, thus $x_0 = -2$</td><td>$V = \pi \left\{ \int_{1/2}^{1} \left[(2+2)^2 - \left(\frac{1}{y} + 2 \right)^2 \right] dy + \int_1^2 \left[(2+2)^2 - (1+2)^2 \right] dy + \int_2^3 \left[(2+2)^2 - (y-1+2)^2 dy \right] \right\} =$
$= \left(15\tfrac{2}{3} - 4\ln 2\right)\pi$</td><td>$V = 2\pi \int_1^2 (x+2)\left[1 + x - \tfrac{1}{x} \right] dx =$
$= \left(15\tfrac{2}{3} - 4\ln 2\right)\pi$</td></tr>
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AKh (06/19/2006-02/09/12)