BRONX COMMUNITY COLLEGE
of the City University of New York

DEPARTMENT OF MATHEMATICS & COMPUTER SCIENCE

MTH 31 Review Sheet

PART I:

1. Explain the concepts of function, domain and range.
2. State the definition of the derivative of a function at a point.
3. Give examples to illustrate that not all continuous functions are differentiable.
4. State the chain rule.
5. State the Intermediate Value Theorem and mention some of its main applications.
6. State the Mean Value Theorem and some of its main applications.
7. Explain how to find local and absolute maxima and minima of a function.
8. State the Fundamental Theorem of Calculus and explain why it is fundamental.
9. Name two mathematicians credited with the invention of Calculus.

PART II:

1. Find the following limits if they exist. If a limit doesn't exist, state so and explain why.

(a) $\displaystyle\lim_{x\to 4}\frac{x^2+9}{x^2-1}$ (b) $\displaystyle\lim_{x\to 4^-}\frac{x-4}{|x-4|}$ (c) $\displaystyle\lim_{\theta\to 0}\frac{\sin(2\theta)}{\tan(\theta)}$

(d) $\displaystyle\lim_{\alpha\to 0}\frac{\alpha}{\cos(\alpha)}$ (e) $\displaystyle\lim_{x\to 0}\sin\frac{1}{x}$ (f) $\displaystyle\lim_{x\to\infty}e^{-x}\sin x$

(g) $\displaystyle\lim_{x\to -3}\frac{|x+3|}{x+3}$ (h) $\displaystyle\lim_{x\to 0}\frac{\sin 5x}{4x}$ (i) $\displaystyle\lim_{x\to 0}\frac{\sin x - x}{x^3}$

(j) $\displaystyle\lim_{x\to 1}\frac{\ln x}{x^2-1}$ (k) $\displaystyle\lim_{x\to 0}\left(\csc x-\frac{1}{x}\right)$ (l) $\displaystyle\lim_{x\to 0^+}x^x$

(m) $\displaystyle\lim_{x\to 0^+}(x\ln x)$ (n) $\displaystyle\lim_{t\to 0}t^2\cos\frac{1}{t}$ (o) $\displaystyle\lim_{x\to\infty}(\ln(x+1)-\ln(x-1))$

(p) $\displaystyle\lim_{x\to 1^-}\frac{2-x}{\ln x}$ (q) $\displaystyle\lim_{x\to 0}x\cot x$ (r) $\displaystyle\lim_{x\to 0^+}\sqrt{x}e^{\cos\frac{\pi}{x}}$

2. Let

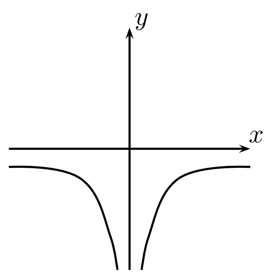
$$f(x)=\begin{cases} ax^2-3x+4 & \text{if } x\le 2\\ x+3a & \text{if } x>2\end{cases}$$

Find the real number a so that f is continuous on $(-\infty,\infty)$.

3. Show that each of the following equations has a solution in the given interval. State clearly what theorem you are using.
   (a) $x^4 - 2x^3 + 3x^2 - 2x - 6 = 0$ in $(-1, 1)$
   (b) $2^x = x^2$ in $(-1, 0)$
   (c) $\cos x = x$ in $\left(0, \dfrac{\pi}{2}\right)$

4. By examining the graphs calculate the required limits. If you think that a certain limit or value doesn't exist state so.

a)



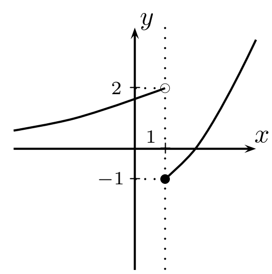
$\lim_{x\to 0^+} f(x) =$

$\lim_{x\to 0} f(x) =$

$\lim_{x\to 0^-} f(x) =$

$f(0) =$

b)



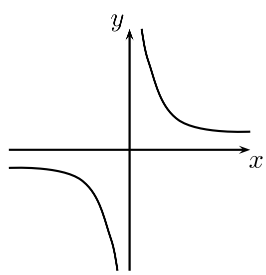
$\lim_{x\to 1^+} f(x) =$

$\lim_{x\to 1} f(x) =$

$\lim_{x\to 1^-} f(x) =$

$f(1) =$

c)



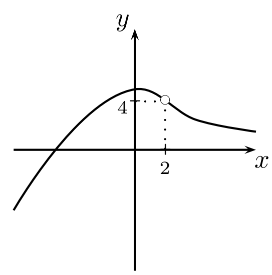
$\lim_{x\to 0^+} f(x) =$

$\lim_{x\to 0} f(x) =$

$\lim_{x\to 0^-} f(x) =$

$f(0) =$

d)



$\lim_{x\to 2^+} f(x) =$

$\lim_{x\to 2} f(x) =$

$\lim_{x\to 2^-} f(x) =$

$f(2) =$

5. Use the definition of the derivative to find $f'(3)$ if $f(x) = x^2 - 2x$.

6. Calculate each of the following derivatives using the definition of the derivative as the limit of the difference quotients:

   A. $\dfrac{d}{dx}\left(x^3 - 3x^2 + 5x - 2\right)$ B. $\left(\dfrac{1}{x^2}\right)'$ C. $\left(\dfrac{x+1}{x+2}\right)'$ D. $\dfrac{d}{dx}\left(\sqrt{2-3x}\right)$

7. Let $f(x) = \sqrt[3]{x}$.
   (a) Use the definition of the derivative to find $f'(x)$ for $x \neq 0$.
   (b) Show that $f'(0)$ doesn't exist.
   (c) What does the calculation in part (b) mean geometrically?

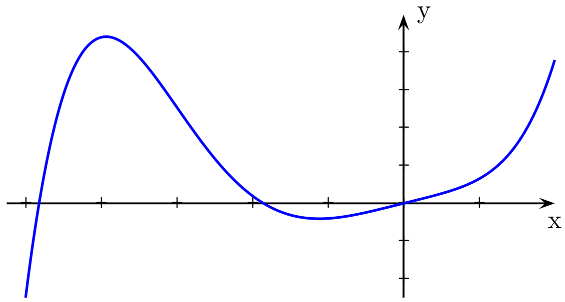
   Hint. *You may want to use the identity*

$$(a-b)(a^2 + ab + b^2) = a^3 - b^3$$

   to "cancel the zeros" in your calculations.

8. Let $f(x) = ax^2 + bx + c$ for constants a, b and c.
   (a) Show that the slope of the tangent to f at $x = x_0$ is $2ax_0 + b$
   (b) Find the equation of the tangent line to the graph of f at $x = x_0$
   (c) Find the equation of the tangent line to the graph of $f(x) = x^2 - x + 16$ at $x = 4$

9. The figure shows the graph of $y = f(x)$. Sketch the graph of $y = f'(x)$.



10. Find an equation of the tangent line to the curve at the given point:
    (a) $y = \dfrac{x+1}{x-1}$, at the point $x = 2$.
    (b) $y = 4\sin^2 x$, at the point $(\frac{\pi}{6}, 1)$.
    (c) $y = \dfrac{x^2-4}{x^2+4}$, at the point $(0, -1)$.
    (d) $y = \sqrt{4 - 2\sin x}$, at the point $(0, 2)$.
    (e) $x^3 + 3x^2y - 2xy^2 - y^3 = 49$, at the point $(3, 2)$.
    (f) $x^{2/3} + y^{2/3} = 4$, at the point $(-3\sqrt{3}, 1)$.
    (g) $y \sin x^2 = x \sin y^2$, at the point $(\sqrt{\pi}, \sqrt{\pi})$.
    (h) $y = e^{2x-1}$ at the point $(\frac{1}{2}, 1)$.
    (i) $y = \ln(2x^2 - x)$ at the point $(1, 0)$.

11. Find $f'(x)$ if

(a) $f(x) = x\tan(x)$	(b) $f(x) = \sin(\tan(2x))$	(c) $f(x) = \left(\dfrac{x+1}{x-1}\right)^2$
(d) $f(x) = x\left(\ln x\right)^3$	(e) $f(x) = \cos\left(\ln x\right)$	(f) $f(x) = \log_2\left(\left(x^2+1\right)^3 \sin(\alpha x)\right)$
(g) $f(x) = \dfrac{x^2}{2\log_3 x}$	(h) $f(x) = 3^{\sqrt{x}}$	(i) $f(x) = \dfrac{1}{\arcsin 2x}$
(j) $f(x) = e^{3x}\sinh x$	(k) $f(x) = \dfrac{(\tanh 3x)(\cosh 2x)}{\sqrt{1+2x}}$	(l) $f(x) = x\arctan 5x - \dfrac{1}{10}\ln\left(1+25x^2\right)$

12. Find $\dfrac{dy}{dx}$ if $\sin(x+y) = \tan(xy)$.

13. Use appropriate linear approximations to estimate the following:
    A. $\sqrt{9.04}$ B. $\sin 0.02$ C. $(1.03)^{-1/3}$ D. $29^{1/5}$ E. $\ln 1.5$.

14. Are the following true or false?

(a) If $f''(x) \geqslant 0$ on (a, b) then $f'(x)$ is increasing on (a, b).

(b) If $f''(x) \geqslant 0$ on (a, b) then $f(x)$ is increasing on (a, b).

(c) If $f''(x_0) = 0$ then x_0 is a point of inflection.

(d) If x_0 is a point of inflection then $f''(x_0) = 0$.

(e) If $f'(x)$ is decreasing on (a, b) then $f(x)$ is concave down on (a, b).

15. A particle moves on a vertical line according to the law of motion

$$s(t) = 2t^3 - 9t^2 + 12t + 3, \qquad t \geq 0$$

where t is measured in seconds and s in meters.

(a) Find the velocity and the acceleration of the particle at time t.

(b) When is the particle moving upward and when is it moving downward?

(c) When is the particle speeding up and when is it slowing down?

(d) Find the total distance traveled by the particle during the first six seconds.

16. Let $f(x) = 3x^4 + 4x^3 - 12x^2 - 10$. Find the (absolute) extremum values of f in the interval $[-3, 2]$.

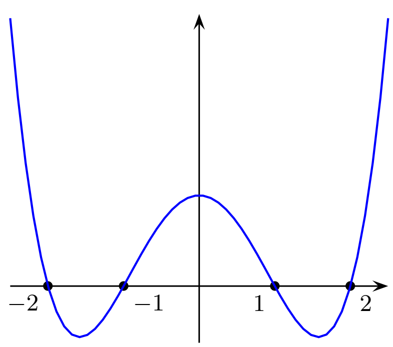
17. Prove that the equation

$$2x^3 + 9x^2 + 42x - 5 = 0$$

has exactly one real solution. State clearly what theorems you are using.

18. The derivative of the function $f(x)$ is $f'(x) = 2(x-1)^2(2x+1)$. Find all critical points of $f(x)$ and determine whether a relative maximum, relative minimum, or neither occurs at each critical point.

19. The graph of the derivative $f'(x)$ of a function $f(x)$ is shown below. The two local minima of $f'(x)$ occur at $x = -3/2$ and $x = 3/2$.



(a) Find the open intervals where f is increasing or decreasing.

(b) Find the open intervals where f is concave up or down.

(c) Where local minima or maxima of f occur?

(d) Where inflection points of the graph of $y = f(x)$ occur?

20. For the function f we have that $f(0) = 3$ and for all x with $0 \leq x \leq 2$ we have that $-5 \leq f'(x) \leq 5$. Use the Mean Value Theorem to find an interval that contains $f(2)$.

21. For each of the following functions:

(a) $f(x) = \sin x - x$, on $[-2\pi, 2\pi]$.

(b) $f(x) = x^3 - x$.

(c) $f(x) = 2 - \dfrac{3}{x} - \dfrac{3}{x^2}$.

(d) $f(x) = \dfrac{x^2 - 4}{x^2 - 1}$.

(e) $f(x) = \dfrac{x^2 - 4}{x^2 + 4}$.

(f) $f(x) = \sec x$.

(g) $f(x) = \dfrac{1 - \cos x}{\sin x}$.

(h) $f(x) = \dfrac{4 + 6x}{\sqrt{x}}$.

(i) $f(x) = e^{-x^2}$.

(j) $f(x) = e^{-x} \sin x$.

(k) $f(x) = x\,(\ln x)^2$.

sketch a graph of the function. The graph should correctly indicate x and y intercepts, local extrema, points of inflection, the intervals where f is increasing or decreasing, the intervals where f is concave upwards or downwards, and any horizontal or vertical asymptotes.

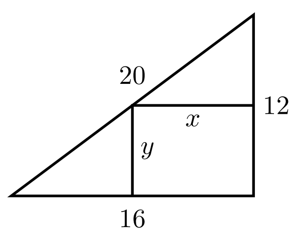
22. A cylindrical cup is 3 inches in diameter. If you're drinking soda from the cup through a straw at a rate of 3 cubic inches per second, how fast is the level of the soda dropping?

23. The surface area of a cube is increasing at a rate of $2\,\mathrm{cm}^2/\mathrm{min}$. How fast is the volume of the cube increasing when the length of the edge is $20\,\mathrm{cm}$?

24. Boat A travels west at a 50 miles per hour and boat B travels north at 60 miles per hour. The two boats are going to collide in 3 hours. At what rate are the two boats approaching each other 1 hour before the collision?

25. A ladder $5\,\mathrm{m}$ long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of $3\,\mathrm{m/s}$, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is $4\,\mathrm{m}$ from the wall?

26. Find the dimensions of the rectangles of maximum area which may be embedded in a right triangle with sides of length 12, 16 and 20 feet as shown in the figure.



27. Find the point in the graph of $y = \sqrt{x+3}$ whose distance from the point $(3, 0)$ is the shortest. What is the shortest distance?

28. Let $f(x) = 2x^3 - 9x^2 + 12x + 5$. Which one is larger $f(1.4562243239684839165)$ or $f(1.4562243239684839167)$? (The two arguments are equal except the very last decimal digit.)

29. On the surface of Mars the acceleration due to gravity is approximately $4\,\mathrm{m/s}^2$. A ball is thrown straight up from a height of $24\,\mathrm{m}$ with an initial velocity of $2\,\mathrm{m/s}$.

    (a) Find an equation for the position function $s(t)$.

    (b) When will the ball reach the highest point? How high will it be then?

(c) When will the ball hit the ground?

(d) How much total distance will the ball cover?

30. Estimate the area under the curve $y = 1/x$ between $x = 1$ and $x = 9$ by using 4 rectangles with heights found at midpoints.

31. Sketch the graph of $f(x) = |x - 1|$, give a geometric interpretation for $\int_{-1}^{2} |x-1|\,dx$ and evaluate this definite integral.

32. Let
$$f(x) = \int_0^x \frac{t^2 + 3t + 2}{t^2 + t + 2}\,dt.$$

(a) Find the open intervals where f is increasing or decreasing.

(b) At what numbers does f attain local extrema?

(c) Find the open intervals where f is concave up or down.

(d) At what numbers does the graph of $y = f(x)$ have inflection points?

33. Evaluate the following:

(a) $\displaystyle\int_{\pi}^{2\pi} \sin\theta\,d\theta$ (b) $\displaystyle\int_{-\pi}^{\pi} t^2 \sin t\,dt$ (c) $\displaystyle\int_1^y \frac{dx}{\sqrt{x}}$ (d) $\displaystyle\int_1^2 \left(x^3 - 2 + \frac{1}{x^2}\right) dx$

(e) $\displaystyle\int \frac{x^2 - 4}{x^{2/3}}\,dx$ (f) $\displaystyle\int x\sqrt{x-5}\,dx$ (g) $\displaystyle\int x^3 \sin(x^4 + 2)\,dx$ (h) $\displaystyle\int \left(x^2 + 8x + \frac{3}{x^2}\right) dx$

(i) $\displaystyle\int \frac{\cos\theta}{\sin^3\theta}\,d\theta$ (j) $\displaystyle\int_1^4 \frac{e^{1/u}}{u^2}\,du$ (k) $\displaystyle\int_1^e \frac{\ln x}{x}\,dx$ (l) $\displaystyle\int (1 + \ln x)\,x^x\,dx$

(m) $\displaystyle\int \frac{9}{t^2+1}\,dt$ (n) $\displaystyle\int \frac{t^2 - 2t}{t^3 - 3t^2 + 7}\,dt$ (o) $\displaystyle\int_0^4 \frac{xe^{\sqrt{x^2+9}}}{\sqrt{x^2+9}}\,dx$ (p) $\displaystyle\int \frac{\sec^2 x}{\tan x}\,dx$

34. Use Newton's Method to find $\sqrt[3]{30}$ correct to four decimal places.

35. Use 3 iterations of Newton's Method to approximate a zero of $f(x) = -x^3 + x + 1$. Set $x_1 = 1.0000$ as the initial guess and round to 4 places after each iteration.

Answers to the exercises in PART II

1. (a) $5/3$, (b) -1, (c) 2, (d) 0, (e) does not exist, (f) 0, (g) does not exist, (h) $5/4$, (i) $-1/6$, (j) $1/2$, (k) 0 , (l) 1, (m) 0, (n) 0, (o) 0, (p) $-\infty$, (q) 1 , (r) 0.

2. $a = 4$.

3. Apply the Intermediate Value Theorem. For (a), set $f(x) = x^4 - 2x^3 + 3x^2 - 2x - 6$, and observe that f is continuous on $[-1, 1]$ that at the endpoints f has opposite signs. Similarly for (b) consider $f(x) = 2^x - x^2$ on $[-1, 0]$, and for (c), $f(x) = \cos x - x$ on $[0, \pi/2]$.

4. a) $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x) = \lim_{x\to 0} f(x) = -\infty$, $f(0)$ is undefined.
b) $\lim_{x\to 1^+} f(x) = -1$, $\lim_{x\to 1^-} f(x) = 2$, $\lim_{x\to 1} f(x)$ does not exist , $f(1) = -1$.
c) $\lim_{x\to 0^+} f(x) = -\infty$, $\lim_{x\to 0^-} f(x) = \infty$, $\lim_{x\to 0} f(x)$ does not exist , $f(0$ is undefined.
d) $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x) = \lim_{x\to 2} f(x) = 4$, $f(2)$ is undefined.

5. $f'(3) = 4$.

6. Calculate $\displaystyle\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ for the appropriate function f.

A. $\dfrac{d}{dx}\left(x^3 - 3x^2 + 5x - 2\right) = 3x^2 - 6x + 5$ B. $\left(\dfrac{1}{x^2}\right)' = -\dfrac{2}{x^3}$ C. $\left(\dfrac{x+1}{x+2}\right)' = \dfrac{1}{(x+2)^2}$

D. $\dfrac{d}{dx}\left(\sqrt{2-3x}\right) = -\dfrac{3}{2\sqrt{2-3x}}$

7. (a) $$\frac{f(x+h) - f(x)}{h} = \frac{\sqrt[3]{x+h} - \sqrt[3]{x}}{h} \cdot \frac{\sqrt[3]{(x+h)^2} + \sqrt[3]{(x+h)x} + \sqrt[3]{x^2}}{\sqrt[3]{(x+h)^2} + \sqrt[3]{(x+h)x} + \sqrt[3]{x^2}} = \frac{(x+h) - x}{h\left(\sqrt[3]{(x+h)^2} + \sqrt[3]{(x+h)x} + \sqrt[3]{x^2}\right)}.$$

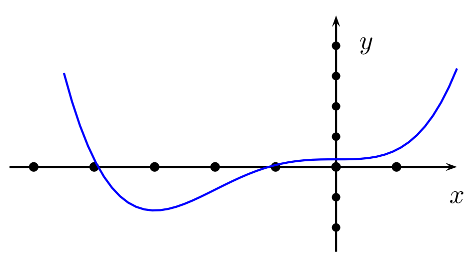
After simplifying and taking limit as $h \to 0$ we find have $f'(x) = \dfrac{1}{3\sqrt[3]{x^2}}$.

(b) After simplifications $\displaystyle\lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0} \frac{1}{\sqrt[3]{h^2}} = \infty$. Thus $f'(0)$ does not exist.

(c) The tangent line of $y = f(x)$ at $x = 0$ is vertical.

8. (b) $y = (2ax_0 + b)x + (c - ax_0^2)$ (c) $y = 7x$

9.



10. (a) $y = -2x + 7$, (b) $y - 1 = 2\sqrt{3}\left(x - \dfrac{\pi}{6}\right)$, (c) $y = 1$, (d) $y = -\dfrac{x}{2} - 2$, (e) $y = \dfrac{55}{9}x - \dfrac{49}{3}$
(f) $y = \sqrt{3}x + 10$, (g) $y = x$, (h) $y = 2x$, (i) $y = 3x - 3$.

11. (a) $f'(x) = \tan(x) + x\sec^2(x)$, (b) $f'(x) = 2\sec^2(x)\cos(\tan(2x))$, (c) $f'(x) = -4\dfrac{x+1}{(x-1)^3}$,
(d) $f'(x) = (\ln x)^3 + 3(\ln x)^2$, (e) $f'(x) = -\dfrac{\sin(\ln x)}{x}$, (f) $f'(x) = \dfrac{1}{\ln 2}\left(\alpha\tan\alpha x + \dfrac{6x}{x^2+1}\right)$,

(g) $f'(x) = \dfrac{x}{\log_3 x} - \dfrac{x}{2\ln 3\,(\log_3 x)^2}$, (h) $f'(x) = \dfrac{3^{\sqrt{x}}\ln 3}{2\sqrt{x}}$, (i) $f'(x) = \dfrac{-1}{2\sqrt{1-x^2}\arcsin^2 x}$,

(j) $f'(x) = e^{3x}\,(3\sinh x + \cosh x)$, (k) $f'(x) = \dfrac{2\cosh 2x}{\sqrt{1+2x}} - \dfrac{\sinh 2x}{(2x+1)\sqrt{1+2x}}$, (l) $f'(x) = \arctan 5x$

12. $\dfrac{dy}{dx} = \dfrac{y\sec^2(xy)}{\cos(x+y) - x\sec^2(xy)}$.

13. A. 3.00667 B. 0.02 C. 0.99 D. 1.4 E. 0.5

14. (a) True (b) False (c) False (d) True (e) True

15.

(a) $v(t) = \dfrac{ds(t)}{dt} = 6t^2 - 18t + 12, \quad a(t) = \dfrac{dv(t)}{dt} = 12t - 18.$

(b) The particle moves forward for $0 < t < 1$ and $t > 2$ and backwards for $1 < t < 2$.

(c) The particle is speeding up for $1 < t < \frac{3}{2}$ (moving backwards) and for $t > 2$ (moving forward). It's slowing down for $0 < t < 1$ (moving forward) and for $\frac{3}{2} < t < 2$ (moving backwards).

(d) 182 m.

16. f attains an absolute minimum of -42 at $x = -2$ and an absolute maximum of 22 at $x = 2$.

17. Set $f(x) := 2x^3 + 9x^2 + 42x - 5$. Use Intermediate Value Theorem to prove that f has at least one zero. Then prove that $f'(x) \neq 0$ for all real values of x and therefore by Rolle's theorem f cannot attain the same value more than once. Conclude that f has exactly one real zero.

18. Solving $f'(x) = 0$ gives the critical points at $x = -1/2,\ 1$. Then $f'(x)$ is negative on the interval $(-\infty, -1/2)$, positive on $(-1/2, 1)$ and positive on $(1, \infty)$. There is a relative minimum of $-27/16$ at $x = -1/2$ using first derivative test and an inflection point at $x = 1$.

19.

(a) From the graph we see that $f'(x)$ is positive for x in the intervals $(-\infty, -2)$, $(-1, 1)$, and $(2, \infty)$. So f is increasing on those intervals. Similarly, f is decreasing on the intervals $(-2, 1)$ and $(1, 2)$.

(b) From the graph we see that f' is decreasing on the intervals $(-\infty, -3/2)$, and $(0, 3/2)$. Therefore f is concave down on those intervals. Similarly, f is concave up on the intervals $(-3/2, 0)$, and $(3/2, \infty)$.

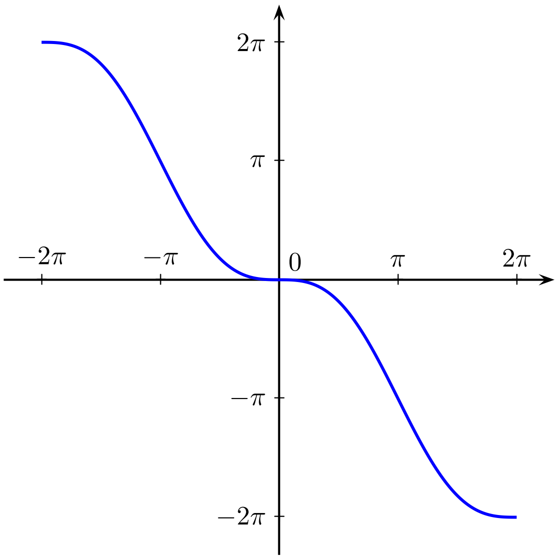
(c) f has local minima at $x = -1$ and $x = 2$, and local maxima at $x = -2$ and $x = 1$.

(d) The graph of $y = f(x)$ has inflection points at $x = \pm 3/2$.

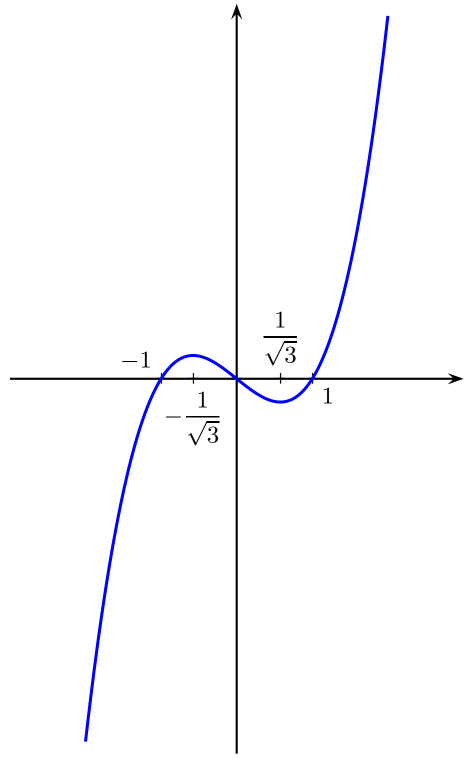
20. By the Mean Value Theorem we have that $f(2) - f(0) = f'(c)(2-0) = 2f'(c)$ for some c in $(0, 2)$. Using that $-5 \leq f'(c) \leq 5$ we conclude $-10 \leq f(2) - 3 \leq 10$ and therefore $-7 \leq f(2) \leq 13$.

21.

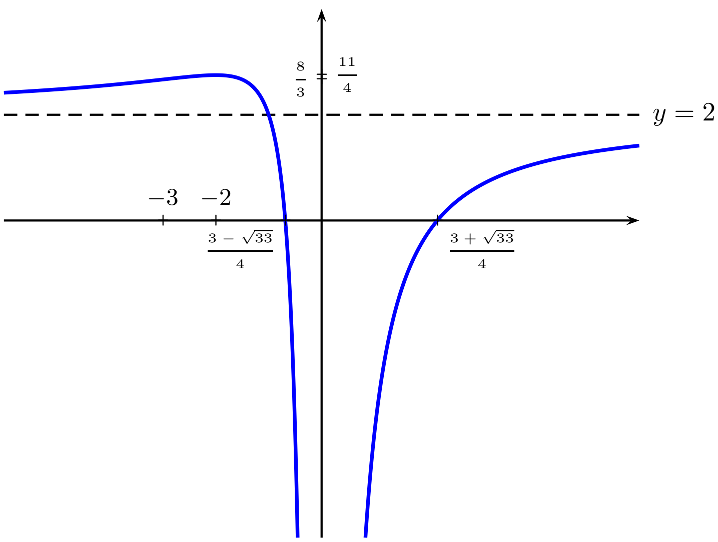
(a)


2π
π
-2π
$-\pi$
0
π
2π
$-\pi$
-2π

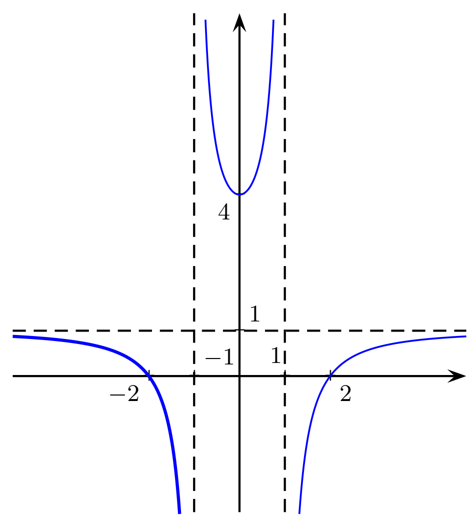
(b)


$\frac{1}{\sqrt{3}}$
-1
$-\frac{1}{\sqrt{3}}$
1

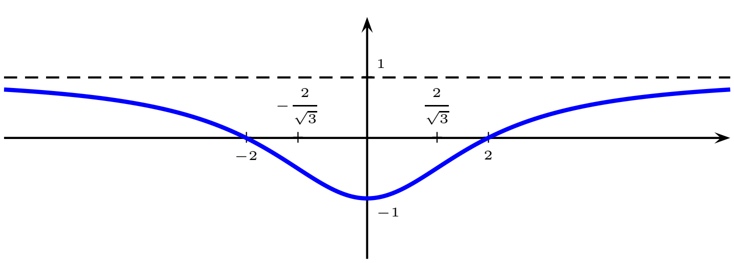
(c)


$\frac{8}{3}$
$\frac{11}{4}$
$y = 2$
-3
-2
$\frac{3-\sqrt{33}}{4}$
$\frac{3+\sqrt{33}}{4}$

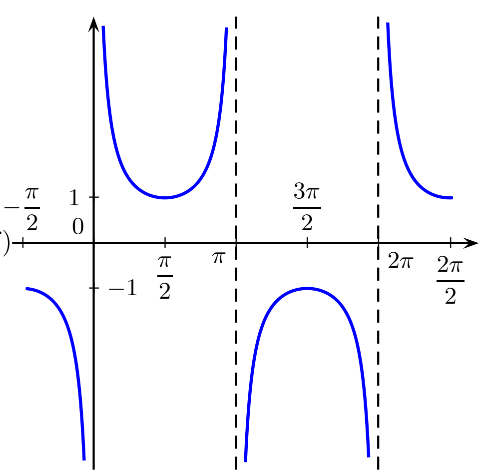
(d)

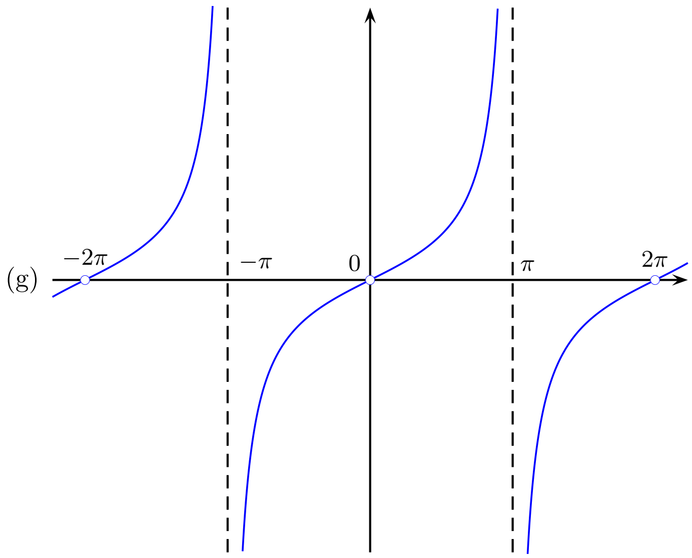

4
1
-1
1
-2
2

(e)

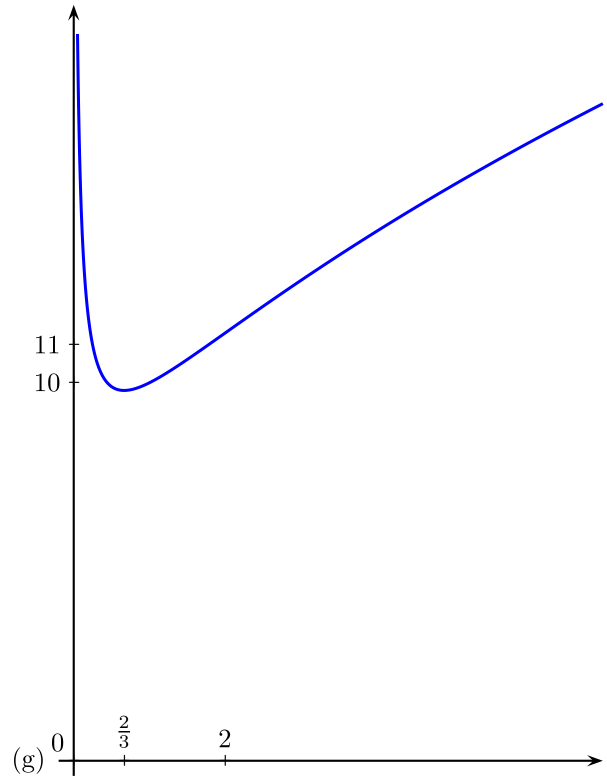

1
$-\frac{2}{\sqrt{3}}$
$\frac{2}{\sqrt{3}}$
-2
2
-1

(f)

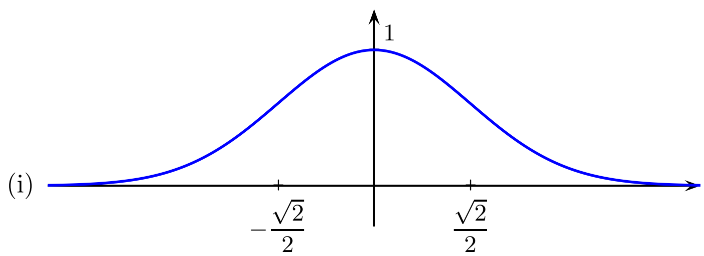

$-\frac{\pi}{2}$
1
0
$\frac{3\pi}{2}$
$\frac{\pi}{2}$
π
2π
$\frac{2\pi}{2}$
-1



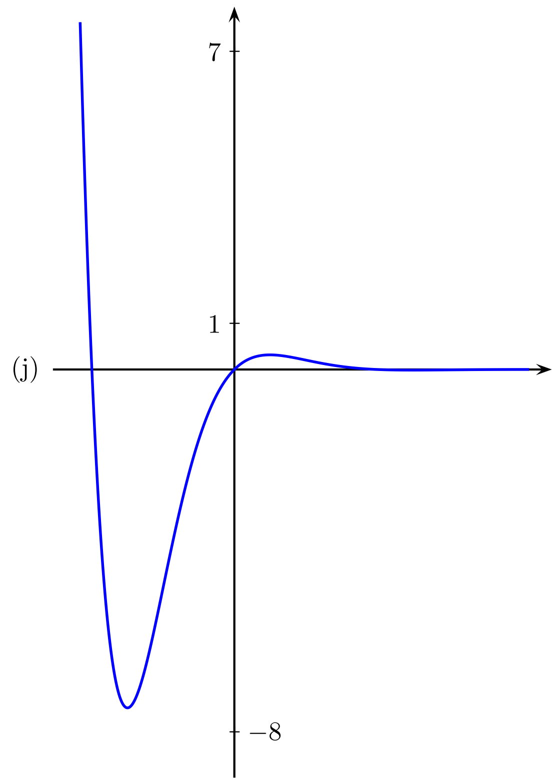
(g)
-2π
$-\pi$
0
π
2π



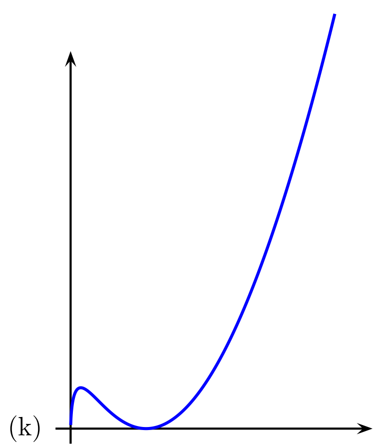
11
10
0
(g)
$\frac{2}{3}$
2



(i)
1
$-\frac{\sqrt{2}}{2}$
$\frac{\sqrt{2}}{2}$



7
1
(j)
-8



(k)

22. The level of the soda is dropping at the rate of 0.42 inches per second.

23. When the length of the edge is 20 cm the volume of the cube increases at a rate of $10\,\text{cm}^3/\text{min}$.

24. One hour before the collision the two boats are approaching each other at a rate of $10\sqrt{61} \approx 78.1$ mi/h.

25. When the bottom of the ladder is 4 m away from the wall the top slides down at a rate of 1.5 m/s.

26. $x = 6$ and $y = 8$ feet.

27. The point is $(5/2, \sqrt{11/2})$ and the shortest distance is $\sqrt{23}/2$.

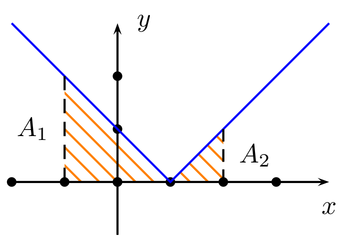
28. Set $a = 1.45622432396848391 65$ and $b = 1.4562243239684839167$. Then $1 < a < b < 2$. By examining f' we see that f is decreasing on $(1, 2)$ and therefore $f(a)$ is largest.

29. (a) $s(t) = -2t^2 + 2t + 24$ (b) The ball reaches the maximum height of 24.5 m after half a second. (c) The ball hits the ground after 4 seconds. (d) 75 m.

30.

$$\Delta x = \frac{9-1}{4} = 2, \quad f(x) = \frac{1}{x}, \quad \sum_{i=1}^{4} f(x_i^*)\Delta x = \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8}\right) 2 = \frac{25}{12}.$$

31.



$$\int_{-1}^{2} |x-1|\, dx = A_1 + A_2 = \frac{1}{2}(2)(2) + \frac{1}{2}(1)(1) = \frac{5}{2}.$$

32.

(a) f is increasing in the intervals $(-\infty, -2)$, and $(-1, \infty)$, and decreasing in $(-2, -1)$.

(b) f attains a local maximum at $x = -2$ and a local minimum at $x = -1$.

(c) f is concave up on the interval $(-\sqrt{2}, \sqrt{2})$, and concave down on the intervals $(-\infty, \sqrt{2})$, and in $(\sqrt{2}, \infty)$.

(d) The graph of $y = f(x)$ has inflection points at $x = \sqrt{2}$, and $x = -\sqrt{2}$.

33. (a) -2, (b) 0, (c) $2\sqrt{y} - 2$, (d) $9/4$, (e) $(3x - 84)\sqrt[3]{x}/7 + C$, (f) $2(x-5)^{5/2}/5 + 10(x-5)^{3/2}/3 + C$, (g) $-\cos(x^4 + 2)/4 + C$, (h) $x^3/3 + 4x^2 - 3/x + C$, (i) $-\csc^2\theta/2 + C$, (j) $e - \sqrt[4]{e}$, (k) $1/2$, (l) $x^x + C$, (m) $\tan^{-1} t + C$, (n) $\ln(t^3 - 3t^2 + 7)/3 + C$, (o) $e^{\sqrt{x^2+9}} + C$, (p) $\ln(|\tan x|) + C$.

34. The recursion is $x_{n+1} = x_n - (x_n^3 - 30)/(3x_n^2)$. x_3 and x_4 agree on the first four decimal places, so $\sqrt[3]{30} \approx 3.1072$.

35. $x \approx 1.3252$.