Bronx Community College of The City University of New York
Department of Mathematics and Computer Science

$$\boxed{\text{Math 23 Review Sheet.}}$$

1. Classify each of the following data according to the *level of measurement* (that is state whether it is nominal, ordinal, interval, or ratio):

   (a) The telephone numbers in a telephone directory.

   (b) The scores of a class in an exam.

   (c) Absolute temperatures (that is temperatures measured in Kelvin degrees).

   (d) Motion Picture Association of America Ratings Description (G, PG, PG-13, R, NC-17).

   (e) Average Monthly precipitation in inches for New York, NY.

   (f) Average Monthly temperature (in degrees Fahrenheit) for New York, NY.

2. A group of 25 people were observed regarding their TV habits and were found to spend the following number of hours per week watching television:

30	32	34	36	36
37	39	39	41	41
42	42	43	43	44
45	45	45	46	47
47	49	49	52	53

   In order to display the data in clearer form,

   (a) determine the class width for four (4) classes,

   (b) construct a frequency distribution showing the class limits for the four classes,

   (c) in the table, show the class boundaries and the class marks,

   (d) construct a histogram, labeling the class boundaries,

   (e) then draw the frequency polygon showing the class marks.

3. A consumer testing service obtained the following mileage (in miles per gallon) in five test runs for three different types of compact cars:

	First Run	Second Run	Third Run	Fourth Run	Fifth Run
Car A	28	32	28	34	30
Car B	31	31	29	29	31
Car C	32	29	28	32	30

   (a) If the manufacturer of Car A wants to advertise that their car performed the best in this test, which "average" (mean, median or mode) should be used to support their claim?

   (b) What "average" should the manufacturer of Car B use to claim that their car performed best?

   (c) What "average" should the manufacturer of Car C use to support a similar claim?

4. Florida's age distribution has mean value $\mu = 39.2$ and standard deviation $\sigma = 24.8$ (measured in years). Use Chebyshev's theorem to find an interval such that

(a) the age in years of at least 75% of Florida's population is contained within that interval,

(b) the age in years of at least 88.9% of Florida's population is contained within that interval,

(c) the age in years of at least 93.8% of Florida's population is contained within that interval.

5. Calculate the range, mean, median, first and third quartiles, interquartile range, mode, variance, and standard deviation for the following data. Draw the box-and-whisker plot.

$$47 \quad 59 \quad 50 \quad 56 \quad 56 \quad 51 \quad 53 \quad 57 \quad 52 \quad 49$$

6. Find the mean, the range and the standard deviation for the following set of sample data.

$$10 \quad 9 \quad 12 \quad 11 \quad 8 \quad 15 \quad 9 \quad 7 \quad 8 \quad 6$$

7. Determine the range and the sample standard deviation of the following data:

| x	f
10	7
20	12
40	5
44	2

8. The mean value of the scores in a Statistics exam was 85 with a standard deviation of 4. Find an interval that contains at least 75% of the scores in that exam.

9. The manager of a salmon cannery suspects that the demand for her product is closely related to the disposable income of her target region. To test out this hypothesis she collected the following data for five different target regions, where x represents the annual disposable income for a region in millions of dollars and y represents sales volume in thousands of cases.

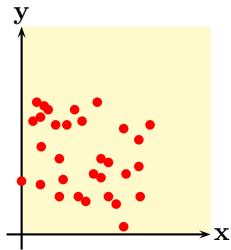| x	y
10	1
20	3
40	4
50	5
30	2
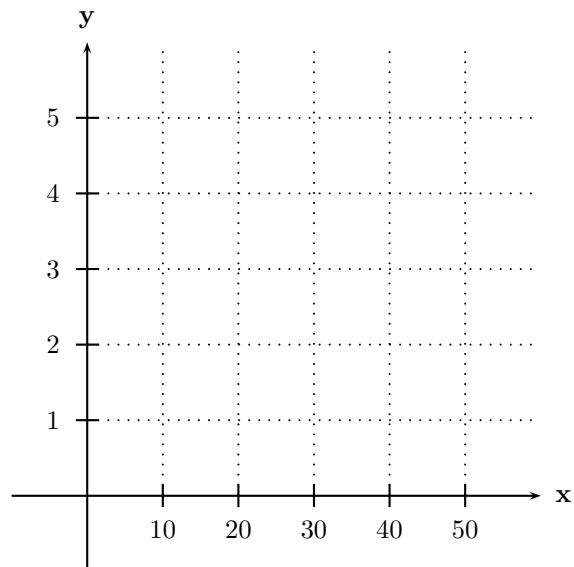
(a) Draw the scatter graph of this set of data.

(b) Compute the correlation coefficient r.

(c) Find and plot the least square line.

(d) If a region has disposable annual income $25,000,000$ what is the predicted sales volume?
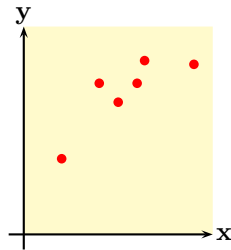
10. Match the appropriate statement about r and the scatter diagrams.
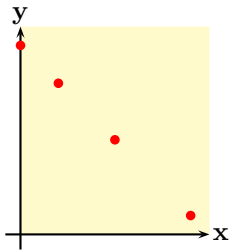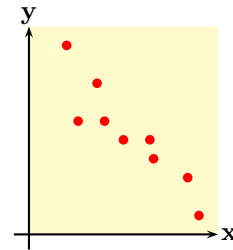

(a)   (b)   (c)   (d)

A. $-1 < r < 0$ B. $r = 0$.   C. $r = -1$.   D. $0 < r < 1$

11. How many three-letter words can be formed of 21 consonants if
    (a) Repetitions are allowed?
    (b) Repetitions are not allowed?

12. Two dice are rolled. Find the probability of the following events:
    (a) Both numbers are 6.
    (b) The first dice gives 5 and the second 6.
    (c) There is one 5 and one 6.
    (d) The sum is equal to 10.
    (e) The sum is equal to 6.
    (f) The sum is 6 or 10.
    (g) The sum is more than 5 but less than 8.
    (h) Both numbers are even.
    (i) One number is even and one number is odd.
    (j) Both numbers are odd.

13. Calculate by hand (without a calculator). Show all work:
    (a) 5!,
    (b) $C(8,5)$.

14. An urn contains three yellow, four green and five blue balls. Two balls are randomly drawn without replacement. Find the probability of the following events:
    (a) Both balls are blue.
    (b) The first ball is green and the second yellow.
    (c) There is one green and one yellow ball.

15. Repeat the previous exercise but now assume that the balls are drawn with replacement.

16. Three cards are randomly drawn from a standard 52 card deck without replacement. Find the probability of the following events:
    (a) All cards are red.
    (b) There are two red and one black card.
    (c) All cards are spades.
    (d) There is one spade, one club, and one diamond.
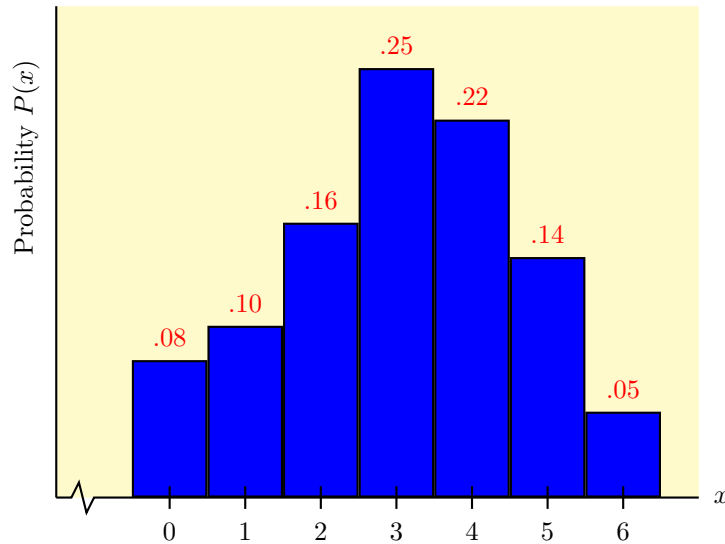
(e) All cards are aces.

(f) Two cards are aces and one card is a King.

17. Consider the following discrete probability distribution:

x	2	3	4	5	6
P(x)	.25	.1	.3	.2	.15

Sketch the graph of this distribution and calculate its expected value and standard deviation.

18. Find the expected value and the standard deviation of the probability distribution whose graph is shown:

19. A fair coin is tossed 7 times. Sketch the graph of the resulting binomial distribution.

20. Alice and Bob play the following game: two cards are randomly drawn (without replacement) from a standard 52-card deck, if they are both red Alice wins otherwise Bob wins. If they play these game 16 times what is the probability that Alice will win at most 4 times?

21. If 30% of the people in a community use the Library in one year, find the probability that in a random sample of 15 people

(a) At most 7 use the Library,

(b) Exactly 7 use the Library,

(c) At least 5 use the Library.

22. A basketball player makes 70% of the free throws he shoots. If he tries 15 free throws what is the probability that he will make more than 7 throws?

23. One-third of all deaths are caused by heart attacks. If three deaths are chosen randomly, find the probability that none resulted from heart attack.

24. Let z have the standard normal distribution. For each of the following probabilities, draw an appropriate diagram, shade the appropriate region and then determine the value:

(a) $P(0 < z < 1.74)$

(b) $P(0.62 \leq z \leq 2.48)$

(c) $P(z \geq 2.1)$

(d) $P(-1.31 < z < 1.07)$.

25. Let z have the standard normal distribution. For each of the following probabilities, draw an appropriate diagram, shade the appropriate region and then determine the value of z_c:

    (a) $P(0 < z < z_c) = 0.4573$

    (b) $P(z_c \leq z \leq 0) = 0.3790$

    (c) $P(z \leq z_c) = 0.1190$

    (d) $P(-z_c \leq z \leq z_c) = 0.8030$.

26. Let x be a normally distributed random variable with $\mu = 70$ and $\sigma = 8$. For each of the following probabilities, draw an appropriate diagram, shade the appropriate region and then determine the value:

    (a) $P(70 \leq x \leq 80.4)$

    (b) $P(61.2 \leq x \leq 85.2)$

27. Find z so that:

    (a) 98% of the area under the standard normal curve lies between $-z$ and z.

    (b) 97.5% of the area under the standard normal curve lies to the left of z.

    (c) 46% of the area under the standard normal curve lies to the right of z.

28. Find the area under the standard normal curve

    (a) between $z = -2.74$ and $z = 2.33$.

    (b) between $z = -2.47$ and $z = 1.03$.

29. The lifetime of a certain type TV tube has a normal distribution with a mean of 80.0 and a standard deviation of 6.0 months. What portion of the tubes lasts between 62.0 and 95.0 months?

30. The scores in a standardized test are normally distributed with $\mu = 100$ and $\sigma = 15$. Find the percentage of scores that will fall below 112.

31. The weights (in pounds) of metal discarded in one week by households are normally distributed with a mean of 2.22 lb. and a standard deviation of 1.09 lb.

    (a) If one household is randomly selected, find the probability that it discards more than 2.00 lb. of metal in a week.

    (b) Find a weight p_{30} so that the weight of metal discarded by 70% of the houses is above x.

32. If the salary of computer technicians in the United States is normally distributed with the mean of $32,550$ and the standard deviation of $2,000$, find the probability for a randomly selected technician to earn more than $35,000$.

33. The weekly amount a family spends on groceries follows (approximately) a normal distribution with mean $\mu = \$200$ and a standard deviation $\sigma = \$15$.

    (a) If $220 is budgeted for next week's groceries what is the probability that the actual cost will exceed the budget?

    (b) How much should be budgeted for weekly grocery shopping so that the probability that the budgeted amount will be exceeded is only 0.05?

34. To determine the mileage of a new model automobile, a random sample of 36 cars was tested. A sample mean of 32.6 mpg with a standard deviation of 1.6 mpg was obtained. Construct the 90% confidence interval for the actual mpg mean of the population of this model automobile.

35. The following data represent a sample of the number of home fires started by candles. Assuming that the number of home fires started by candles is approximately normally distributed find a 95% confidence interval for mean number of home fires started by candles each year.

$$5400 \qquad 5860 \qquad 6070 \qquad 6210 \qquad 7360 \qquad 8450 \qquad 9960$$

36. A random sample of 41 NBA players gave a standard deviation $s = 3.32$ inches for their height. How many more NBA players have to be included in the sample to make 95% sure that the sample mean $\bar{x}$ of their height is within 0.75 inch of the mean μ of the height of the population of all NBA players.

37. A teacher has developed a new technique for teaching which he wishes to check by statistical methods. If the mean of a class test turns out to be 60 (or less), the results will be considered unsuccessful. Alternatively, if the mean is greater than 60, the results will be considered successful. The results of the test with a class of 36 students had a mean $\bar{x} = 66.2$ with a standard deviation of $s = 24.0$. Test whether the results were successful at the $\alpha = 5\%$ level of significance. (Use 1-tail test.) State the null and the alternate hypothesis and include diagrams.

Answers

1. A. Nominal.   B. Ratio.   C. Ratio.   D. Ordinal.   E. Ratio.   F. Interval.

2. The class width has to be 6. We then have the following frequency table.

Class Limits Lower-Upper	Class Boundaries Lower-Upper	Frequency	Class Marks (midpoints)
$30 - 35$	$29.5 - 35.5$	3	32.5
$36 - 41$	$36.5 - 41.5$	7	38.5
$42 - 47$	$41.5 - 47.5$	11	44.5
$48 - 53$	$48.5 - 53.5$	4	50.5

And we have the following histogram and frequency polygon:
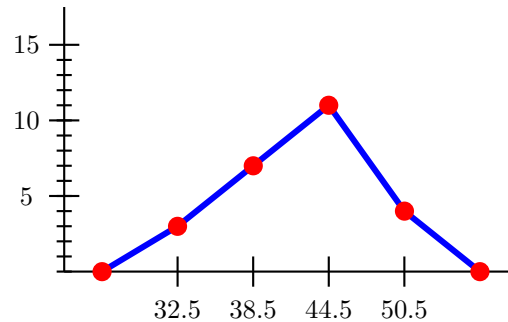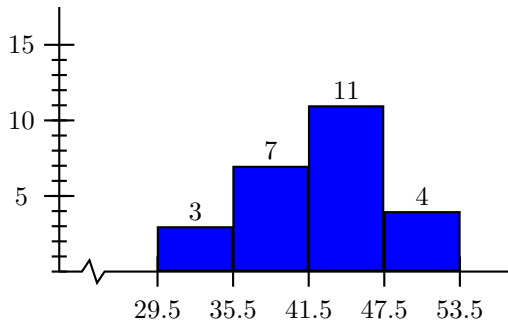
3. A. Mean.   B. Median.   C. Mode.

4. A. $[0, 88]$ B. $[0, 113.6]$ C. $[0, 138.4]$

5. The range is 12, the mode is 56, the mean is $\mu = 53$, the standard variation is $\sigma = 3.69$, the variance is $\sigma^2 = 13.6$. The quartiles are $Q_1 = 50$, the median $Q_2 = 52.5$, and $Q_3 = 56.5$ while the interquartile range is 6. The box-and-whisker plot is shown in Figure 1.

6. Mean is $\bar{x} = 9.89$, range is 8, sample standard deviation is $s = 2.47$.

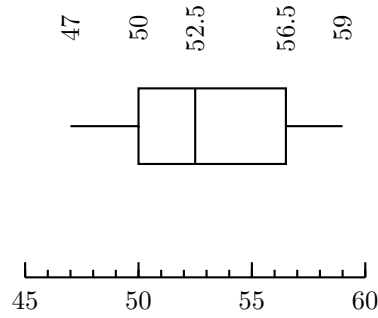7. The range is 34. The standard deviation is $s = 6.03$.

Figure 1: The box-and-whisker plot of problem 5

8. $[77, 93]$

9. The correlation coefficient is $r = 0.9$. The line of least squares is $y = .3 + .9x$. For a region with disposable annual income of $\$25,000,000$ the model predicts sale of $2,550$ cases. The scatter graph and the plot of the line are shown in Figure 2.
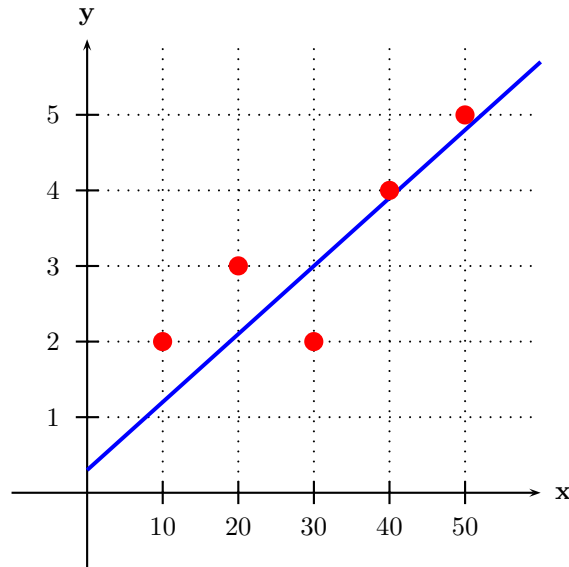

Figure 2: The scatter plot and the regression line of problem 9

10. A. (d)   B. (b)   C. (a)   D. (c)

11. A. 9261   B. 7980.

12. A. $\dfrac{1}{36}$ B. $\dfrac{1}{36}$ C. $\dfrac{1}{18}$ D. $\dfrac{1}{12}$ E. $\dfrac{5}{36}$ F. $\dfrac{2}{9}$ G. $\dfrac{11}{36}$ H. $\dfrac{1}{4}$ I. $\dfrac{1}{2}$ J. $\dfrac{1}{4}$

13. A. 120   B. 56.

14. A. $\dfrac{5}{33}$ B. $\dfrac{1}{11}$ C. $\dfrac{2}{11}$.

15. A. $\dfrac{25}{144}$ B. $\dfrac{1}{12}$ C. $\dfrac{1}{6}$.

16. A. $\dfrac{2}{17}$ B. $\dfrac{13}{34}$ C. $\dfrac{11}{850}$ D. $\dfrac{169}{1700}$ E. $\dfrac{1}{5525}$ F. $\dfrac{3}{850}$.

17. The expected value of the distribution is $\mu = 3.9$ and the standard deviation is $\sigma = 1.37$. The graph of the distribution is
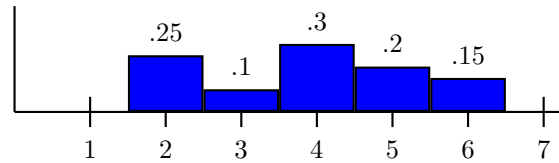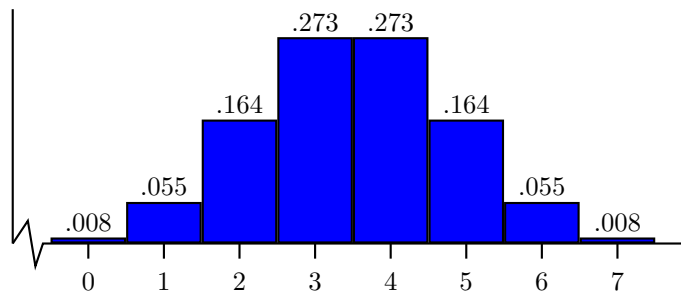

Figure 3: The graph of the probability distribution of Problem 17

18. The expected value is $\mu = 3.05$ and the standard deviation $\sigma = 1.58$.

19. First compute the probabilities (you can also get these values from the tables in the appendix of the textbook):

x	0	1	2	3	4	5	6	7
P(x)	.008	.055	.164	.273	.273	.164	.055	.008

The graph is:

20. $P(0 \le r \le 4) \approx .63$.

21. A. $P(0 \le r \le 7) = .951$ B. $P(r = 7) = .081$ C. $P(5 \le r \le 15) = .485$

22. $P(7 < r \le 15) = .951$. Why is this answer the same as the answer for 21 (a)?

23. $P(r = 0) = \dfrac{8}{27}$.

24. A. 0.4591   B. 0.2610   C. 0.0179   D. 0.7626

25. A. $z_c = 1.72$ B. $z_c = -1.17$ C. $z_c = -1.18$ D. $z_c = 1.29$

26. A. 0.4032   B. 0.8356

27. A. $z = 2.33$ B. $z = 1.96$ C. $z = 0.1$

28. A. 0.987   B. 0.8417

29. 99.24%.

30. 78.81%.

31. A. $P(x > 2.00) = .58$ B. 1.65 lb.

32. 0.11.

33. A. .09   B. $224.67.

34. $[32.16, 33.04]$.

35. $[5518.54, 8570.04]$.

36. 35 more players need to be included.

37. **Partial solution:** $H_0 : \mu = 60$ (or $\mu \leq 60$), $H_a : \mu > 60$. The critical z-value is $z_c = 1.645$. Then
$$z = \frac{66.2 - 60.0}{\frac{24.0}{\sqrt{36}}} = \frac{6.2}{4.0} = 1.55 < z_c$$

Conclusion: do not reject H_0. The results were statistically unsuccessful at the 5% level of significance. (That is the results could not be distinguished from a random sample from a normal population with mean $\mu = 60$ and standard deviation $\sigma = 24.0$.)

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