

Bronx Community College of The City University of New York
Department of Mathematics and Computer Science

Review Sheet for MTH 21, A Mathematical World.

Sets and Counting

1. Let S be the set $\{1, 2, 4, 6, 7, 9\}$.
 - (a) Explain the terms: set, element, subset, cardinality.
 - (b) Is $4 \in S$?
 - (c) Is $\{4, 5\} \subseteq S$?
 - (d) Find $n(S)$, the cardinality of S .
2. Let $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$ be the universal set with $A = \{2, 5, 6, 7\}$, $B = \{0, 1, 2, 5, 7\}$.
 - (a) Find the union: $A \cup B$
 - (b) Find the intersection: $A \cap B$
 - (c) Find A' , the complement of A .
3. For the same sets as in question 2:
 - (a) Draw their Venn diagram representation, showing where each number goes. Make sure to label U , A and B .
 - (b) Find: $(A \cap B)'$
 - (c) Find: $A' \cup B'$
4. If $n(A) = 75$, $n(B) = 42$ and $n(A \cap B) = 27$ then compute $n(A \cup B)$.
5. A survey of 150 people found that 70 owned a bike, 35 owned a scooter and 15 owned both.
 - (a) Draw a Venn diagram for these survey results.
 - (b) How many own a scooter but not a bike?
 - (c) How many own a scooter or a bike?
 - (d) What percentage of the people surveyed own neither?
6. Draw a Venn diagram to represent each of the following:
 - (a) $A \cup B'$
 - (b) $A' \cap B'$
 - (c) $A \cap B \cap C$
7. A serial number consists of two letters followed by a U or a V and then four digits with no digit repeated. For example, a possible serial number here is TQV2930. What is the total number of possible serial numbers?

8. Thirty women run in the olympic marathon final. How many ways can the gold, silver and bronze medals be awarded?
9. All possible permutations of the first 11 letters of the alphabet are printed and all printed pages are collected in binders. If 100 permutations can be printed on one page and each binder contains 200 pages, how many binders are needed?
10. Compute the factorial, permutation and combination numbers: $5!$, ${}_4P_3$, ${}_7C_3$
11. How many ways are there to select 4 people from 20 job applicants if
 - (a) the order of selection is not important,
 - (b) the order of selection is important?
12. Are the following sets A and B equivalent?

$$A = \{2, 4, 6, 8, \dots, 102\}$$

$$B = \{5, 10, 15, 20, \dots, 210\}$$

13. Define the infinite sets $S = \{2, 4, 6, 8, \dots\}$, $T = \{5, 10, 15, 20, \dots\}$.
 - (a) Are S and T equivalent? Explain.
 - (b) What does it mean for an infinite set to be countable?
 - (c) Are S and T countable or uncountable? Explain.
 - (d) Give an example of an uncountable set.

Statistics

14. A survey asked 25 people how many others they lived with. The results were

2	0	1	1	3	2	0	1	0	0	3	0	4
1	2	0	0	1	1	2	0	1	2	1	1	

- (a) Make a frequency distribution table showing frequency and relative frequency.
 - (b) Give the 68th percentile and the 96th percentile for this data set.
 - (c) Draw a histogram.
 - (d) Draw a pie chart.
15. The police checked speeds on a highway and got these results for 20 cars (in mph):

80	56	60	72	51	50	78	63	55	81
85	69	56	61	79	52	53	68	57	66

- (a) Group the data into 4 categories and make a frequency distribution table showing frequency and relative frequency.
- (b) What percentage of drivers were going over 67 mph?
- (c) Draw a histogram with 4 bars.

16. Find the mean, median and mode for this data set:

3 6 2 10 7 4 11 7

17. Sonya scored 83, 75 and 72 on three exams. What score does she need on the fourth exam so that her average is 80?

18. A survey asked 50 people how many films they had watched in the last week. The frequency distribution was

number of films	frequency
0	8
1	15
2	11
3	10
4	6

(a) Compute the mean, median and mode for this data set.

(b) Find the 46th percentile and the 88th percentile for films watched in the last week.

19. Suppose you scored at the 20th percentile on a test. Does that mean you did well or badly? Explain.

Number Theory

20. Use the sieve of Eratosthenes to find all the prime numbers between 1 and 50.

21. Give the prime factorization of 594.

22. Is the number 367 prime or composite? Explain.

23. Check if the following numbers are abundant, perfect or deficient: 6, 12, 81

24. The Fibonacci numbers begin with $F_1 = 1$, $F_2 = 1$, $F_3 = 2$, $F_4 = 3, \dots$. Compute this sequence up to F_{16} .

25. Use Binet's formula

$$F_n = \frac{\phi^n - 1/(-\phi)^n}{\sqrt{5}} \approx \frac{\phi^n}{\sqrt{5}}, \quad \text{with the golden ratio } \phi = \frac{1 + \sqrt{5}}{2},$$

to compute the seventeenth Fibonacci number F_{17} . Check that your answer is the sum of F_{15} and F_{16} from the previous question.

Probability

26. Suppose the probability of rain tomorrow is 0.8. Convert this to a percent and a fraction in lowest terms. Is it likely or unlikely to rain tomorrow?

27. In a roulette game you bet \$5 on the number 2. The house odds are 35 to 1 for this bet. Explain what you win or lose, depending on where the ball finishes up.

28. Roll a die. Let E be the event that you roll a 3 or a 4. Find the probability of this event and the odds of this event.
29. You are dealt a random card from a pack of 52.
- (a) What is the probability that this card is a diamond?
 - (b) What is the probability that this card is the queen of hearts?
 - (c) What is the probability that you get either an ace or a king?
 - (d) What is the probability that you get either a jack or a heart?
 - (e) Of these four events, which is most likely?
30. A jar contains jellybeans with only 4 red, 7 black and 9 yellow beans. You pick a bean without looking.
- (a) What is the probability that it is yellow?
 - (b) What are the odds that it is red?
 - (c) What is the probability that it is not red?
31. (a) An urn contains 25 balls and 3 of them are white. What is the probability that a randomly chosen ball from the urn is white?
- (b) You are in a class with 24 other students. The instructor will collect the homework of three randomly selected students. What is the probability that your homework will be collected?
32. In a clinical trial of a drug, 140 out of 400 patients found it cured their condition. Estimate the probability of the drug curing a similar patient?
33. Two dice are rolled. Let E be the event that sum of the numbers is 6 or less. Let F be the event that you roll a double (both dice the same number). Compute
- (a) $p(E)$
 - (b) $p(F)$
 - (c) $p(E')$, the probability that E does not occur
 - (d) $p(E \cap F)$
 - (e) $p(E \cup F)$
34. Suppose the probability of winning a hamper in a raffle draw is 0.1 and the probability of winning a bottle of wine is 0.2. Also the probability of winning both is 0.05. Find the probability that you win a hamper or a bottle of wine.
35. You are dealt a five card poker hand. What is the probability of getting the ace, king, queen, jack and ten, with all of them hearts? Write the answer as a fraction.
36. What is the probability of being dealt exactly three jacks in a hand of 5 cards? Write the answer as a percentage.
37. A lottery uses balls numbered 1 to 45. In the weekly draw, five numbers are randomly picked from a rotating drum.

- (a) Find the probability that the five numbers on your ticket match and you win the grand prize.
 - (b) Was winning the grand prize more or less likely than a one in a million chance?
 - (c) Find the probability that four of your numbers match and you win a smaller prize.
 - (d) How many times more likely is matching 4 to matching all 5?
38. You are dealt a five card poker hand. What is the probability of getting all spades? Write your answer as an accurate decimal approximation.

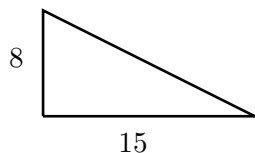
Financial Mathematics

39. Find the simple interest earned on \$4700 invested for 10 years at $2\frac{1}{2}\%$.
40. If \$10 000 is invested for 8 years earning 3% simple interest, what is it worth at the end?
41. Your credit card charges 20% interest. For a billing cycle June 15 through July 14 you have a balance of \$300 at the beginning and pay off \$190 on June 25. On July 4 you make a \$130 purchase on the card. Compute the card's finance charge.
42. Suppose \$4000 earns 6% annual interest, compounded monthly for 11 years. Show that the value at the end is nearly double.
43. How much money should be invested in a savings account now if it is to be worth \$20 000 in 8 years? The account earns 4% interest compounded weekly.
44. If an account earns 6.7% interest compounded daily, what is its annual yield? In other words, what yearly simple interest rate is it equivalent to? Give your answer as a percent rounded to the nearest hundredth of a percent.
45. Carla makes monthly payments of \$150 to an annuity earning 4% interest for 10 years.
- (a) What is the annuity worth at the end?
 - (b) Give Carla's total contribution.
 - (c) Find the total interest.
46. Find the monthly payments to an annuity earning 6% interest for 5 years so that it is worth \$4000 at the end.
47. You buy a car for \$23 700 and obtain a loan for the full amount. The bank charges 15% interest.
- (a) What are the monthly payments to pay off the car in 9 years?
 - (b) Make an amortization schedule for the first two payments. Show each total payment, the principal portion, the interest portion and the balance.
48. You take out a \$350 000 mortgage at 8% interest for 30 years. What are your monthly payments?

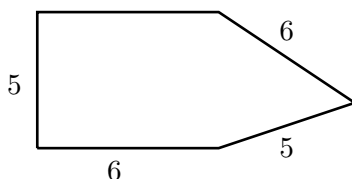
Geometry

49. For a circle of radius 5 meters find, as an accurate decimal in the correct units, its circumference, its diameter and its area.

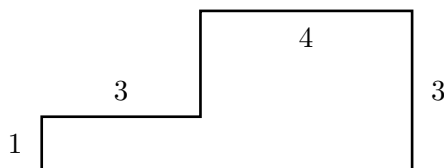
50. Find the area and perimeter of this right triangle, with units given in inches. Your answers must be in the correct units.



51. Find the perimeter and area of this polygon. The units are centimeters.



52. Find the perimeter and area of this polygon. The units are miles.



53. Starting with an equilateral triangle, draw the first steps of the Koch snowflake fractal.
54. (a) Draw a rectangle of length 2 and width 1. If you make it four times bigger, how many copies of the original rectangle fit inside?
- (b) Use the formula $s^d = n$ to compute the dimension d of the original rectangle (the scaling factor s is 4 and n is the number of copies).
- (c) What is unusual about the dimensions of fractal shapes?

Linear Programming

55. A corner point P of a feasible region is the intersection point of the following lines:

$$3x - 4y = 8$$

$$x + 2y = 6.$$

Graph these lines. What are the coordinates of P ?

56. Explain in words how to tell which side of a line to shade when graphing the solution to a linear inequality. Use complete sentences.
57. (a) Graph the triangular region bounded by the inequalities:

$$x \geq 0$$

$$y \geq 0$$

$$3x + 5y \leq 15$$

(b) Give the coordinates of its three corner points.

(c) Maximize the objective function $z = 2x + 3y$ subject to these constraints.

58. A carpenter makes x tables and y chairs from a supply of 300 kilograms of wood every week. Suppose she has time to make at most 15 items per week, and each table needs 40 kg and each chair 12 kg of wood. Give the 4 inequalities that describe this situation.

59. Maximize $z = x + y$ subject to the constraints:

$$\begin{aligned} x &\geq 0 \\ y &\geq 0 \\ x + 4y &\leq 14 \\ 3x + 2y &\leq 12 \end{aligned}$$

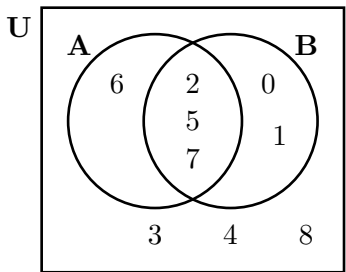
60. Minimize $z = 4x + 5y$ subject to the constraints:

$$\begin{aligned} x &\geq 0 \\ y &\geq 0 \\ x + y &\geq 4 \\ 2x + y &\geq 6 \end{aligned}$$

Answers

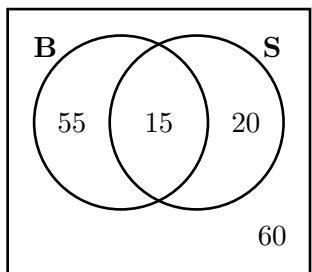
1. (a) See the course text, (b) Yes, (c) No, (d) 6.

2. (a) $\{0, 1, 2, 5, 6, 7\}$, (b) $\{2, 5, 7\}$, (c) $\{0, 1, 3, 4, 8\}$.



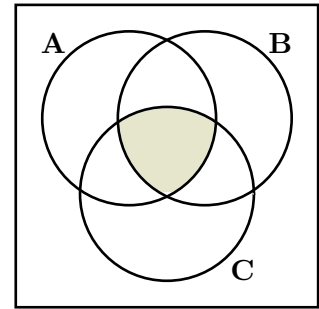
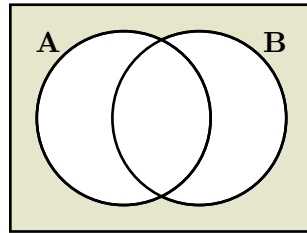
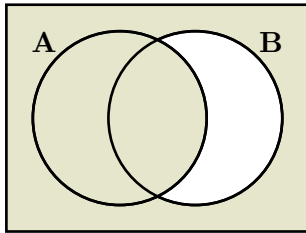
3. (a) (b) $\{0, 1, 3, 4, 6, 8\}$, (c) $\{0, 1, 3, 4, 6, 8\}$.

4. $n(A \cup B) = 90$



B is the set of those owning a bike
S is the set of those owning a scooter

5. (a) (b) 20, (c) 90, (d) 40%.

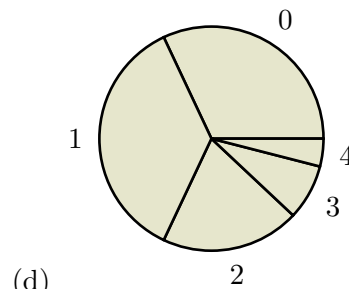
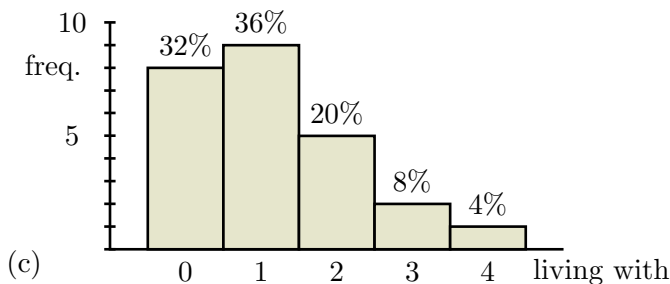


6. (a) (b) (c)
7. $26 \cdot 26 \cdot 2 \cdot 10 \cdot 9 \cdot 8 \cdot 7 = 6\,814\,080$ serial numbers.
8. $30 \cdot 29 \cdot 28 = 24\,360$ ways the medals can be awarded.
9. 1996 binders.
10. (a) $5! = 120$, (b) ${}_4P_3 = 24$, (c) ${}_7C_3 = 35$.
11. (a) 4845, (b) 116280.
12. We can set up a correspondence between A and B with n in A corresponding to $5n/2$ in B . The last element of A is 102 and this corresponds to 255 which is not the last element of B . That means that these sets are not equivalent.
13. (a) The sets S and T are equivalent. Use the correspondence of the previous question. (b) An infinite set is countable if its elements can be put in a one-to-one correspondence with the counting numbers in $N = \{1, 2, 3, 4, \dots\}$. In other words, it is equivalent to N . (c) Yes, S and T are both countable. An element n of S corresponds to $n/2$ in N . An element n of T corresponds to $n/5$ in N . (d) The set of real numbers is an example of an uncountable set.

14. (a)

living with	freq.	rel. freq.
0	8	32%
1	9	36%
2	5	20%
3	2	8%
4	1	4%

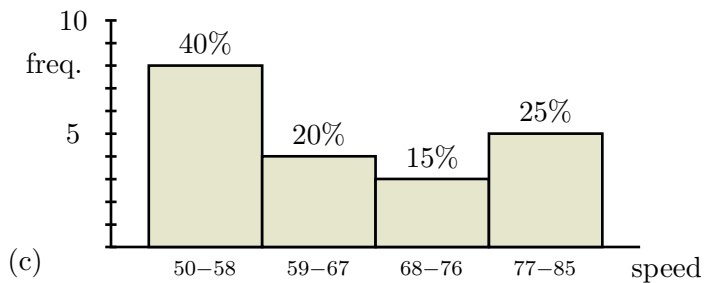
(b) The 68th percentile is 2 and the 96th is 4.



15. (a)

speed	freq.	rel. freq.
50 – 58	8	40%
59 – 67	4	20%
68 – 76	3	15%
77 – 85	5	25%

(b) 40% were going over 67 mph.



- (c)
16. The mean is 6.25, the median is 6.5 and the mode is 7.
 17. Sonya needs 90 on the fourth exam.
 18. (a) The mean is 1.82, the median is 2 and the mode is 1. (b) The 46th percentile is 2 and the 88th percentile is 4.
 19. If you scored at the 20th percentile then that means that 80% of people taking the test did as well or better than you. So you did badly relative to the others.
 20. The primes under 50 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43 and 47.
 21. $594 = 2 \cdot 3^3 \cdot 11$
 22. 367 is prime because none of the primes up to 19 are factors.
 23. 6 is perfect, 12 is abundant and 81 is deficient.
 24. The sequence we want ends with $F_{15} = 610$ and $F_{16} = 987$.
 25. The approximation gives $\phi^{17}/\sqrt{5} \approx 1596.99987$. So $F_{17} = 1597$ and this does equal $610 + 987$.
 26. This is 80% and $4/5$ as a fraction. It is very likely to rain tomorrow.
 27. If your number doesn't come up then you lose your \$5. If your number does come up then you get your \$5 back plus your winnings of \$175.
 28. The probability is $1/3$ and the odds are 1 : 2.
 29. (a) $1/4$, (b) $1/52$, (c) $2/13$, (d) $4/13$, (e) part (d) is the most likely.
 30. (a) $9/20 = 0.45$, (b) 1 : 4, (c) $4/5 = 0.8$.
 31. (a) 0.12, (b) 0.12.
 32. 35%
 33. (a) $5/12$, (b) $1/6$, (c) $7/12$, (d) $1/12$, (e) $1/2$.
 34. 0.25
 35. It is $\frac{1}{{}_{52}C_5} = \frac{1}{2598960}$.

36. The probability is (rounded to the nearest hundredth of a percent)

$$\frac{{}_4C_3 \times {}_{48}C_2}{{}_{52}C_5} = \frac{4512}{2598960} = 0.001736\dots = 0.17\%$$

37. (a) $\frac{1}{{}_{45}C_5} = \frac{1}{1221729}$, (b) less likely, (c) $\frac{{}_5C_4 \times {}_{40}C_1}{{}_{45}C_5} = \frac{200}{1221729}$, (d) 200 times more likely.

38. The probability is $\frac{{}_{13}C_5}{{}_{52}C_5} = \frac{1287}{2598960} \approx 0.000495$.

39. \$1175

40. \$12400

41. The finance charge is \$3.63

42. After 11 years the value is \$7726.45

43. \$14524.77

44. 6.93%

45. (a) \$22087.47, (b) \$18000, (c) \$4087.47.

46. The monthly payments are \$57.33

47. (a) \$401.11,

	payment number	principal part	interest part	total payment	balance
(b)	0	-	-	-	23700.00
	1	104.86	296.25	401.11	23595.14
	2	106.17	294.94	401.11	23488.97

48. The monthly mortgage payments are \$2568.18

49. The circumference is 31.42 m, the diameter is 10 m and the area is 78.54 m².

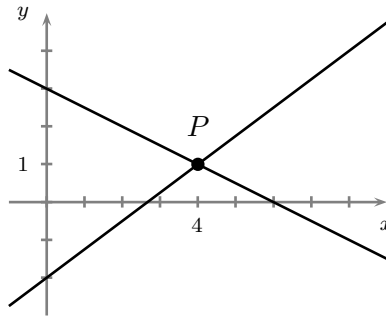
50. The area is 60 square inches and the perimeter is 40 inches.

51. The perimeter is 28 cm and the area is 42 cm² using Heron's formula.

52. The perimeter is 20 miles and the area is 15 square miles.

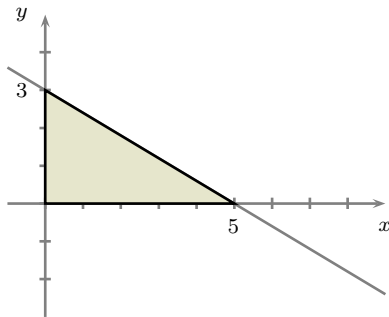
53. See the course text. Divide each side into three equal segments and put an equilateral triangle on the middle segment. Then remove that middle segment. For step 2, repeat this on all the new sides. Repeat again for step 3 ...

54. (a) 16 copies, (b) $4^d = 16$ so the dimension d of the rectangle must be 2, (c) fractals can have dimensions that are not whole numbers.



55. The coordinates of P are $(4, 1)$.

56. See the course text. One method is as follows. Choose a point on one side of the line, (not on the line). The origin $(0, 0)$ is a good choice if it is not on the line. Test the coordinates of your chosen point in the inequality. If they make the inequality true then shade in the side of the line containing your point; all these shaded points are solutions. If they make the inequality false then shade in the other side of the line.



57. (a)

(b) corners: $(0, 0)$, $(5, 0)$ and $(0, 3)$, (c) 10.

58. The inequalities are:

$$\begin{aligned} x &\geq 0 \\ y &\geq 0 \\ x + y &\leq 15 \\ 40x + 12y &\leq 300 \end{aligned}$$

59. The maximum value of z is 5.

60. The minimum value of z is 16.