

WORKBOOK. MATH 21. SURVEY OF MATHEMATICS I.

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

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LESSON 1.

- (1) A child counts as follows: "... twenty-five, twenty-six, twenty-seven, twenty-eight, twenty-nine, twenty-ten, twenty-eleven..." Is this incorrect? Why?

- (2) What do you mean by unit's place, ten's place or hundred's place?

- (3) How would you read 1,348?

- (4) What is the decimal system? (Think about the term dec).

- (5) Explain the number 17 in decimal system.

- (6) Instead of using groups of tens, use groups of 5 to explain the number 17.

- (7) What do you think is the meaning of $17_{10} = 32_5$?

- (8) While counting, why do we go to one-hundred after ninety-nine, instead of saying ninety-ten, or tenty? What does a hundred signify? Or a thousand? What is the next significant jump?
- (9) Now think about how you will use groups of 5 to explain the number 27. This is what it means to work in base 5.
- (10) Use base 7 to explain the number 27.
- (11) Write 152 as a numeral in base 2.
- (12) Write 152 as a numeral in base 5.
- (13) Write 152 as a numeral in base 8.
- (14) Write 152 as a numeral in base 12.

(15) Write 152 as a numeral in base 16.

(16) Convert 123_4 to decimal form.

(17) Convert 123_5 to decimal form.

(18) What is the meaning of the number 0.345_7 ?

(19) What is the meaning of the number 0.345_6 ?

(20) Convert the numeral 0.345_6 to decimal form.

(21) Convert the numeral 78.34_9 to decimal form.

(22) Convert the numeral 39.45_{12} to decimal form.

Practice problems

(1) Fill in the blanks (You will need to do a lot of scratch-work in your notebooks):

Base	Numbers			
Binary				
Quintary			432_5	
Octal				
Decimal	38	39		
Duodecimal				
Hexadecimal				$1E_{16}$

(2) What is wrong with the following number: 78_8 ? How would you correct it?

(3) Convert the numeral 101.111_2 to decimal form.

(4) Convert the numeral $ABC.D_{16}$ to decimal form.

(5) Convert the numeral 666.666_8 to decimal form.

LESSON 2.

(1) What is a set? Give five examples.

(2) What is an element? What do the symbols \in and \notin mean? Explain using five examples.

(3) What is an empty set? How is an empty set denoted?

(4) Is $\{\phi\}$ an empty set? Explain in your own words.

(5) Describe five sets in **set builder notation** (also called **descriptive method**).

- (6) Describe if possible, the five sets from the previous problem in **listing method**.
- (7) What is the **cardinal number** (also called **cardinality**) of a set? Explain using five examples of finite sets and five examples of infinite sets.
- (8) When is a set B called a **subset** of a set A ? What is the notation for describing a subset? Give five examples of subsets of sets.
- (9) Which set is a subset of every set?
- (10) Suppose A and B are two sets such that $A \subseteq B$ and $B \subseteq A$. What can you say about A and B ?
- (11) When is a set B called a **proper subset** of a set A ? What is the notation for describing a proper subset? Give five examples of proper subsets of sets.

(12) Is ϕ a proper subset of every set?

(13) Important examples of infinite sets:

- The **set of natural numbers** = $\mathbb{N} = \{ \quad \quad \quad \}$.
- The **set of whole numbers** = $\mathbb{W} = \{ \quad \quad \quad \}$.
- The **set of integers** = $\mathbb{Z} = \{ \quad \quad \quad \}$.
- The **set of rational numbers** = $\mathbb{Q} = \{ \quad \quad \quad \}$.

(14) When are two sets **equal**? Give five examples of equal sets.

(15) When are two finite sets **equivalent**? Give five examples of equivalent finite sets.

(16) How is equivalence different from equality? Explain using an example.

(17) What is a one-to-one correspondence between two sets? Give five examples of one-to-one correspondence between finite sets and five examples of one-to-one correspondence between infinite sets.

(18) Suppose A and B are two sets and $B \subset A$. Can A and B be equal? Can A and B be equivalent?

(19) Two sets are said to be equivalent if _____.

(20) A set is said to be countably infinite if _____.

(21) Give five examples of countably infinite sets.

Practice Problems

(1) Write the set of odd whole numbers less than 15 in

- listing method

- descriptive method

(2) Write the set of odd integers between -12 and 16 in

- listing method

- descriptive method

(3) Let $A = \{a, b, c, d\}$, $T = \{1, 2, 3, 4, 9, 10\}$, and $S = \{1, a, 2, b, 3, 4, 9, 10, c, d\}$. State true or false:

$A \subseteq T.$	$A \subseteq S.$	$T \subseteq S.$
$A \subset T.$	$A \subset S.$	$T \subset S.$
$S \subset A.$	$\phi \subseteq S.$	$a \in A.$
$a \in T.$	$a \in S.$	$a \notin A.$
$a \notin T.$	$a \notin S.$	$\phi \in S.$

(4) Show that the set $\{0, 3, 6, 9, 12, 15, \dots\}$ is countably infinite. Describe the one-to-one correspondence mathematically.

- (5) Show that the set $\{2, 5, 8, 11, 14, 17, \dots\}$ is countably infinite. Describe the one-to-one correspondence mathematically.
- (6) Show that the set $\{1, 4, 9, 16, 25, 36, \dots\}$ is countably infinite. Describe the one-to-one correspondence mathematically.
- (7) Show that the set $\{1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \frac{1}{36}, \dots\}$ is countably infinite. Describe the one-to-one correspondence mathematically.

LESSON 3.

Basic Law of counting : Suppose that we are asked to choose two items. If the the first item has n choices, and the second item has m choices then there are $n \times m$ choices for a pair of two items.

- (1) From a standard deck of cards I choose one queen and one king. How many choices do I have?

- (2) We throw a pair of dice. How many outcomes do we get?

- (3) From a stack of 6 textbooks, 7 notebooks and 10 pens, I choose one textbook, one notebook and 1 pen. How many choices do I have?

Counting with replacement :

- (1) From a standard deck of cards, I choose one queen. I replace her back. I then again choose a queen. How many choices do I have?

- (2) How many 4-lettered words can be made using the letters a,b,c,d,and e?

- (3) How many 3 digit numbers can be made using the digits 4,5, and 6?

Counting without replacement : Two main types – one where order matters, and the other where order does not matter.

Factorial : The mathematical symbol ! is read **factorial**.

$$1! = 1$$

$$2! = 2 \times 1 =$$

$$3! = 3 \times 2 \times 1 =$$

$$4! = 4 \times 3 \times 2 \times 1 =$$

$$5! = 5 \times 4 \times 3 \times 2 \times 1 =$$

For natural number n , $n! =$

By definition $0! = 1$.

- (1) Jane is planning her vacation. She wants to visit Athens, Bonn, and Chicago. If she does not visit any city more than once, how many different itineraries can she choose from?

- (2) A restaurant has one choice of salad, two choices of main-course and three choices of dessert and four choices of drink. If a meal consists of a salad followed by a main-course and ended with a dessert with a drink on the side, then how many different meals are possible at this restaurant?

- (3) How many five letter words can be formed from the letters a,b,c,d,e if no letter is repeated?

- (4) How many six digit numbers can be made from the digits 2,3,4,5, and 9 if we do not repeat any of the digits?

Permutations A permutation of r objects is their arrangement where **order matters**.

Number of permutations of r objects taken from n objects

$$P_{n,r} = \frac{n!}{(n-r)!} \quad \text{for natural numbers } n, r \text{ where } r \leq n.$$

Note : Alternative notations for $P_{n,r}$ are ${}^n P_r$ or, P_r^n , or $P(n, r)$.

$$P_{4,2} =$$

$$P_{5,3} =$$

$$P_{7,4} =$$

- (1) Jane is planning her vacation. She wants to visit two cities among Athens, Bonn, Chicago and Detroit. If she does not visit any city more than once, how many different itineraries can she choose from?
- (2) How many three letter words can be formed from the letters a,b,c,d,e if no letter is repeated?
- (3) How many four digit numbers can be made from the digits 1,2,3,4,5,6, and 9 if we do not repeat any of the digits?

Combinations A combination of r objects is their arrangement where **order does not matter**.

Number of combinations of r objects taken from n objects

$$C_{n,r} = \frac{n!}{(n-r)!r!} \quad \text{for natural numbers } n, r \text{ where } r \leq n.$$

Note : Alternative notations for $C_{n,r}$ are nC_r or, C_r^n , or $C(n, r)$.

$$C_{4,2} =$$

$$C_{5,3} =$$

$$C_{7,4} =$$

- (1) Jane is planning her vacation. She wants to visit two cities among Athens, Bonn, Chicago and Detroit. If she does not visit any city more than once and does not worry about which city she visits first, how many different itineraries can she choose from.

- (2) In how many ways can three letters be chosen from the letters a,b,c,d,e if no letter is repeated?

- (3) In how many four digits be chosen from the digits 1,2,3,4,5,6, and 9 if we do not repeat any of the digits?

Practice problems

- (1) In how many ways can four cards be dealt from a standard deck of cards?

- (2) A committee of three members is to be formed from an office consisting of thirty members. How many committees are possible if no person occupies two positions?

- (3) A committee of three members is to be formed from an office consisting of thirty members. The committee consists of a chair, an assistant chair, and a finance-secretary. If no person occupies two positions, then how many committees are possible?

- (4) Find the number of ways to answer ten multiple-choice questions if each question has five choices.

- (5) Fifteen people are at a party. If each person at the party shakes the hand of everyone else at the party, then determine the total number of handshakes.

- (6) How many seven-digit phone numbers are possible if a phone number does not start with a 1 or a 0?

- (7) How many anagrams can be made from the word **mississippi**?

LESSON 4.

Probability of the occurrence of an event is the ratio of the number of ways that the event can occur to the total number of outcomes. Let E denote an event and $P(E)$ denote the probability of the occurrence of event E . Then

$$P(E) = \frac{\text{number of ways event } E \text{ can occur (desired)}}{\text{total number of outcomes (total)}}$$

(1) A card is drawn from a standard deck of cards. What is the probability that

- a ten-card is drawn?

- a diamond card is drawn?

- a face-card (J,Q,K) is drawn?

- a number card (A, 2, ... 10) is drawn?

(2) A pair of dice is thrown. What is the probability that

- two sixes show up?

- the sum of the numbers that show up is 7?

- the sum of the numbers that show up is at least 7?

- the sum of the numbers that show up is at most 7?

Complement of an event E is the nonoccurrence of event E . Complement of event E is denoted by \overline{E} . Some properties of probability:

$$0 \leq P(E) \leq 1; \quad P(\overline{E}) = 1 - P(E)$$

(1) A card is drawn from a standard deck of cards. What is the probability that

- a ten-card is not drawn?

- a diamond card is not drawn?

- a face-card (J,Q,K) is not drawn?

- a number card (A, 2, ... 10) is not drawn?

(2) A pair of dice is thrown. What is the probability that

- two sixes do not show up?

- the sum of the numbers that show up is not 7?

- the sum of the numbers that show up is not at least 7?

- the sum of the numbers that show up is not at most 7?

Practice problems

(1) Two cards are drawn at random from a standard deck of cards. What is the probability that

- both the cards are 10?

- one card is a 10 and the other is a 9?

- both the cards are face cards?

- they are not both 5?

- one card is a face and the other is not a 10?

(2) If there is a 65% chances of you catching the flu this season, then what is the chance that you do not catch the flu this season?

(3) A coin is flipped 5 times. What is the probability that at least 1 head shows up?

LESSON 5.

More probability calculations.

- (1) If three lettered words are made from the letters a,b,c,d,e, what is the probability that the first letter is a?

- (2) If 7 cards are drawn from a standard deck of cards, then what is the probability that
 - all are hearts?

 - two are Kings, three are Queens and 2 are Jacks.

- (3) If three dice are thrown, then what is the probability that
 - the sum of the numbers which come up is at least 16?

 - at least one 3 shows up?

- (4) From a class of 20 students, two students are chosen each day to present seminar lectures over a period of 10 days. If you are a member of this class, then
 - what is the probability of you being chosen the first day?

 - what is the probability of you being chosen the seventh day, if you have not been chosen for the first six days?

LESSON 6.

Odds : The **odds** for any event is a ratio of the number of desired outcomes to the number of outcomes that are not desired.

From probability to odds : If E denotes occurrence of an event and $P(E)$ denotes the probability that E occurs, then the odds for event E is given by

$$O(E) = \frac{P(E)}{P(\bar{E})}.$$

(1) The probability of an event is 35% . What are the odds of the event?

(2) If there is a 85% chance that you will wake up when the alarm rings, then

- what is the probability that you will not wake up when the alarm rings?

- what are the odds of you getting up when the alarm rings?

- what are the odds of you not getting up when the alarm rings?

(3) The probability of an event is $\frac{2}{3}$. What are the odds of the event?

From odds to probability : If the odds for an event E is given by the ratio $a : b$ then

$$P(E) = \frac{a}{a + b}.$$

(1) If the odds of an event E are 4 to 3, then find the probability of the occurrence of event E .

(2) If the house odds for winning the prize in an Iowa lottery are posted as 3,400:1.

- What is the probability of winning the lottery?

- What are the odds of winning the lottery?

- How much should you win if a lottery ticket costs \$ 5?

(3) If the house odds of a game are 9:1, then what is the probability that the house wins? What is the probability that you win?

LESSON 7.

Expected value is the expected winnings from a contest or a game.

- (1) Suppose that you play a game with a pair of dice, which costs \$ 1 to play. If you get no six on the top, you get no prize. If you get one six on the top, then you get \$ 5. If you get two sixes then you get \$ 10. What is the expected winnings of this game?

- (2) Suppose you play a game of flipping a coin three times. To play this game you are charged \$ 1. If you get 0,1, or, 2 heads then you get no prize. If you get 3 heads you get \$ 10. What is the expected winnings in this game?

Summary : To find the expected value of a game, find the sum of the products of the probability of each event and the amount won or lost if that event occurs.

A game is called **fair** if the expected value of the game is zero.

- (1) Are the two previous games fair?

- (2) Suppose that a certain game costs \$ 3 to play. The probability of winning is 0.3. How much should you win for the game to be fair?

- (3) Suppose that a certain game costs \$ 2 to play. Suppose that the net prize when you win is \$ 8. The only possible outcomes of the game are winning and losing. What is the probability of winning if the game is fair?

- (4) Suppose that a certain contest costs \$ 1 to play and has the following odds and prizes. Is the contest fair?

Odds	Prize
1 to 99	\$ 5
1 to 999	\$ 10
1 to 9,999	\$ 100

LESSON 8.

Let us start with a short review of some topics from coordinate geometry.

Slope of a line passing through two points (x_1, y_1) and (x_2, y_2) is

$$m = \text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}.$$

(1) Find the slope of a line passing through the given points. In each case explain what the slope means.

- $(1,2)$ and $(2,5)$.

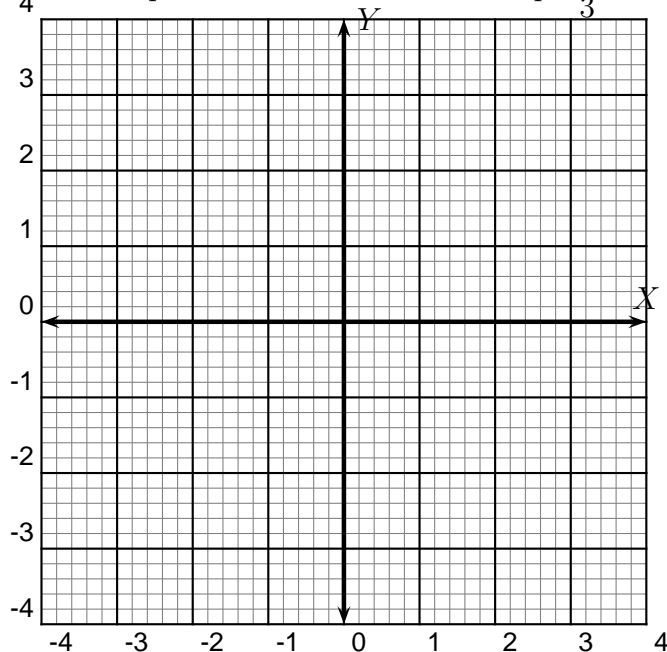
- $(1,-2)$ and $(1,5)$.

- $(-1,-4)$ and $(2,-3)$.

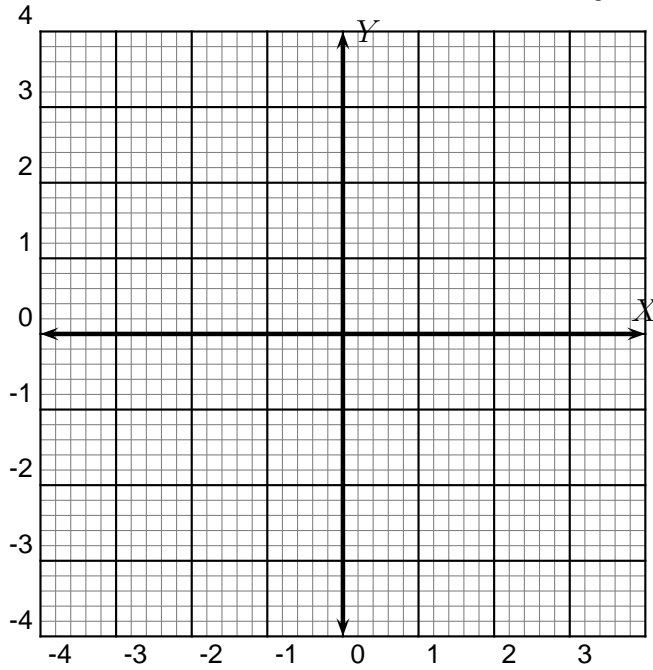
- $(-1, -4)$ and $(2, -4)$.

Equation of a line : The **slope-intercept** equation of a line is $y = mx + b$ where m is the slope and b is the y-intercept of the line.

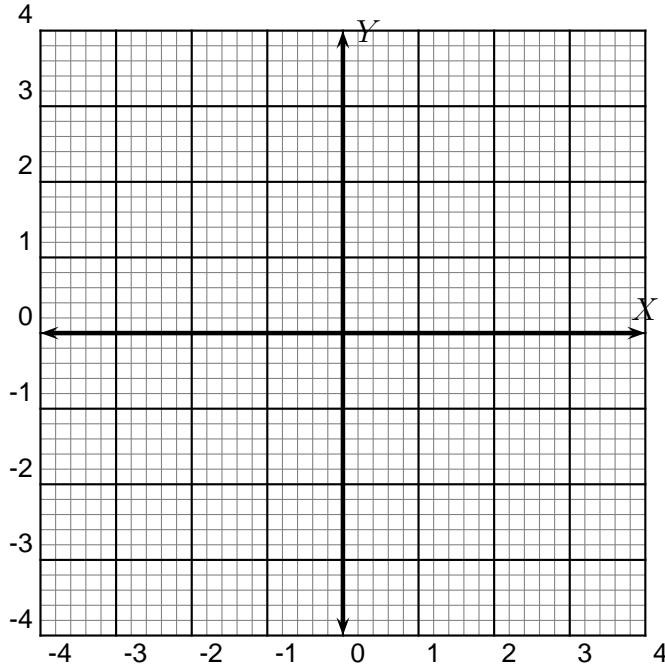
(1) Give an equation of a line that has slope $\frac{2}{3}$ and y-intercept -3 . Draw the line.



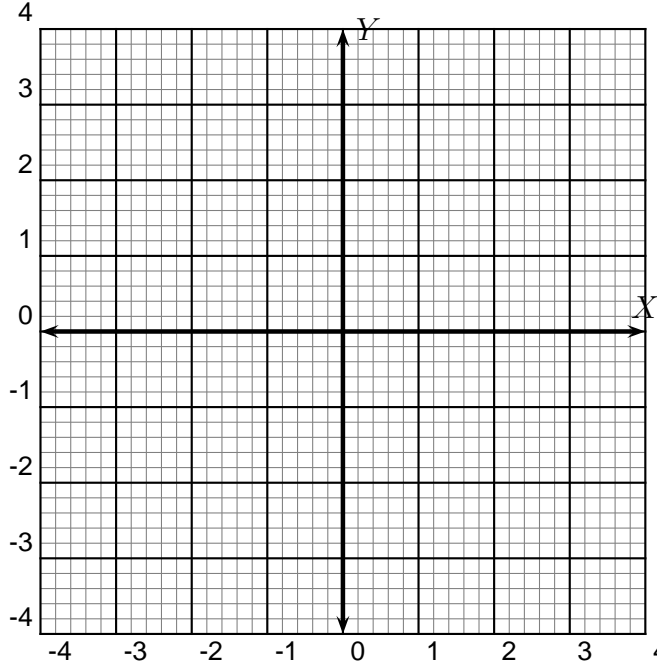
- (2) Give an equation of a line that has slope $-\frac{1}{3}$ and passes through $(1, 2)$. Draw the line.



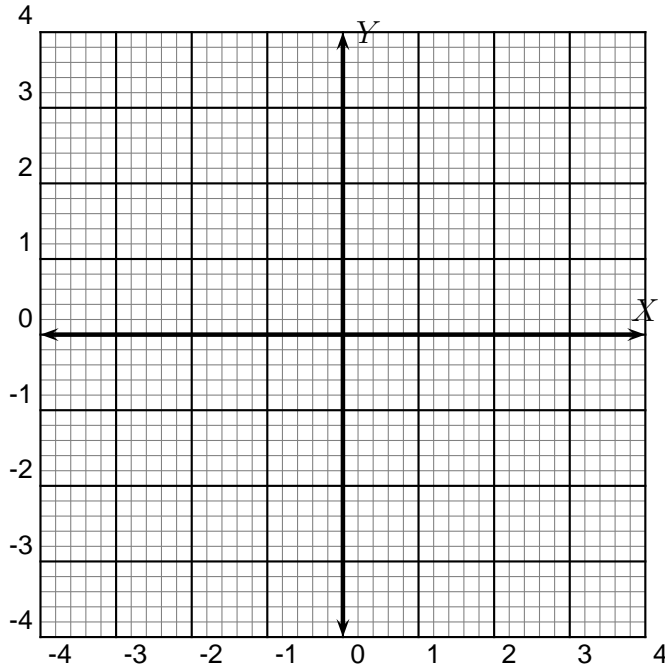
- (3) Give an equation of a line passing through points $(-1, 2)$ and $(2, 3)$. Draw the line.



- (4) Draw the line given by the equation $2x + 3y = 6$.

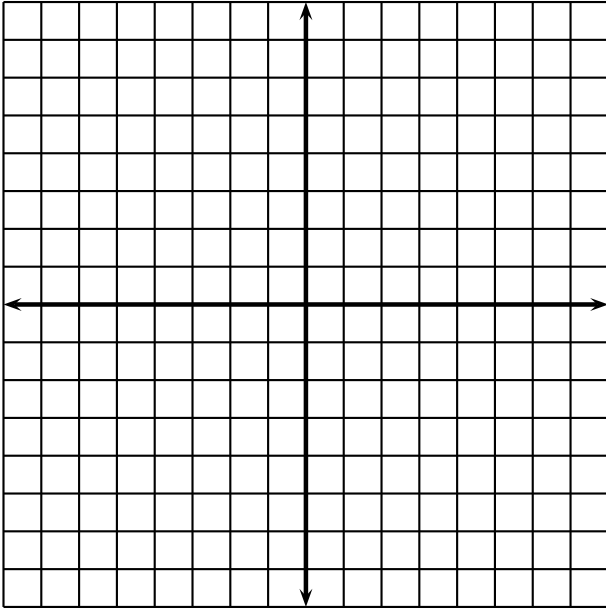


- (5) Draw the line given by the equation $3x + 5y = 15$.

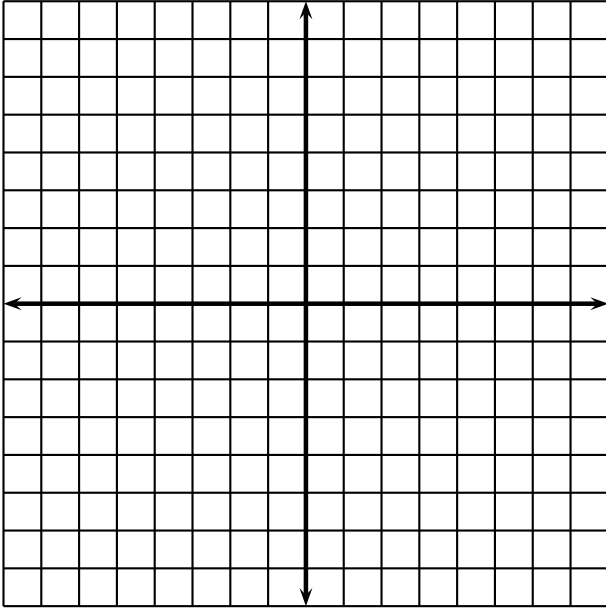


End of review.

- (1) Graph the equation $s = 2.5t$. Name the axes appropriately.



- (2) Graph the equation $a = \frac{45}{47}v$. Name the axes appropriately.



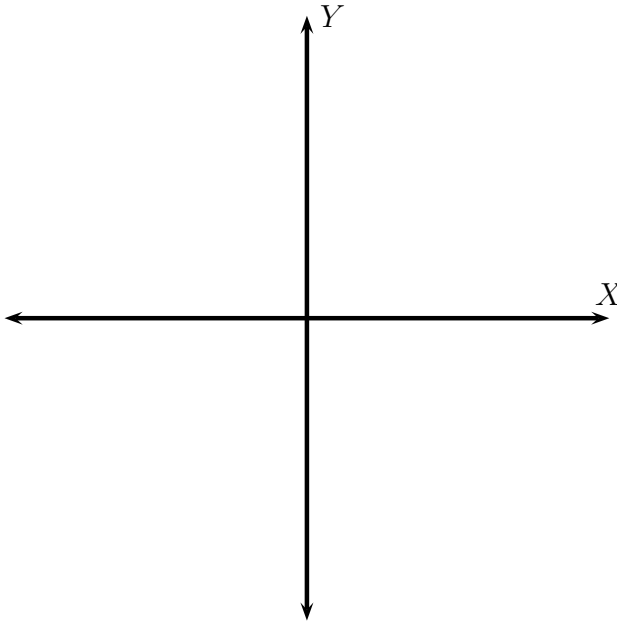
- (3) Find five integer solutions to the equation $3x - 2y = 2$.

- (4) Suppose you own a factory that prints notebooks. The cost (C) in dollars of printing x (in hundreds) of notebooks is given by the equation $C = 45x + 300$.
- What does the number 300 signify?
 - What does the number 45 signify?
 - Find the cost of printing 600 books.
 - How many books may be printed if you are allowed to spend \$ 1200?
- (5) A particular car is purchased for \$ 35,000. A year later, the car is worth only \$ 30,650. If the value of the car depreciates at the same rate, determine how long it will take the car to depreciate to \$ 8,900.

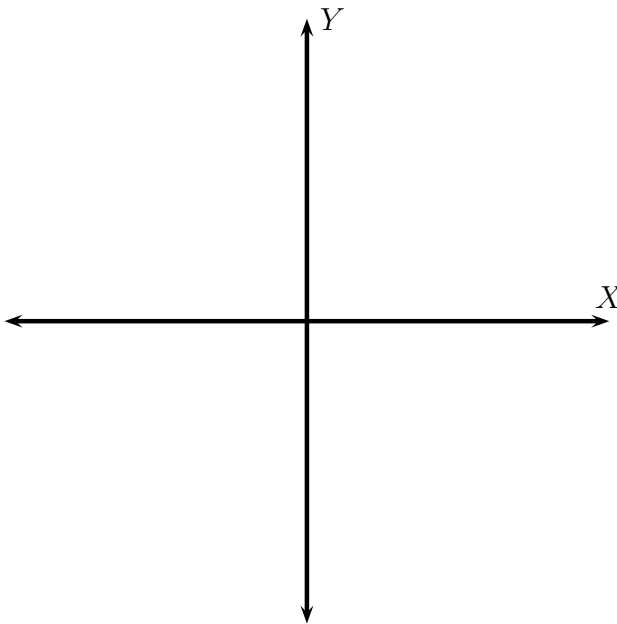
LESSON 9.

Review of quadratic equations.

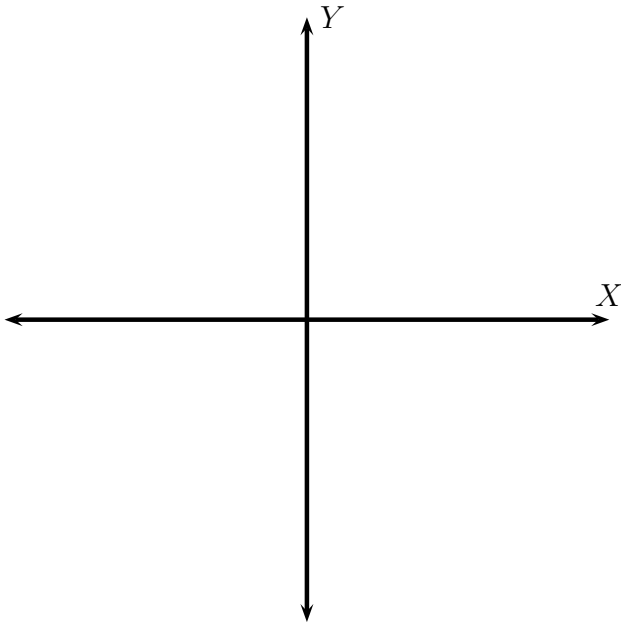
- (1) Draw the curve given by the equation $y = x^2$. What is the vertex? Does the curve open up or open down?



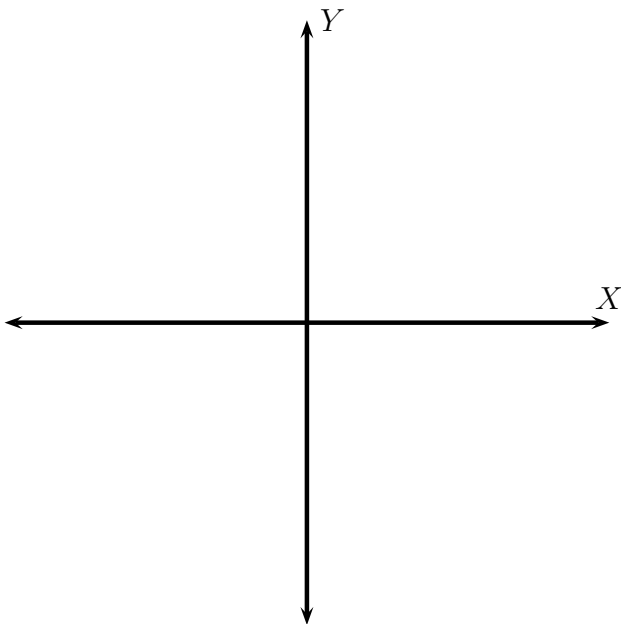
- (2) Draw the curve given by the equation $y = -x^2$. What is the vertex? Does the curve open up or open down?



- (3) Draw the curve given by the equation $y = (x + 3)^2$. What is the vertex? Does the curve open up or open down?

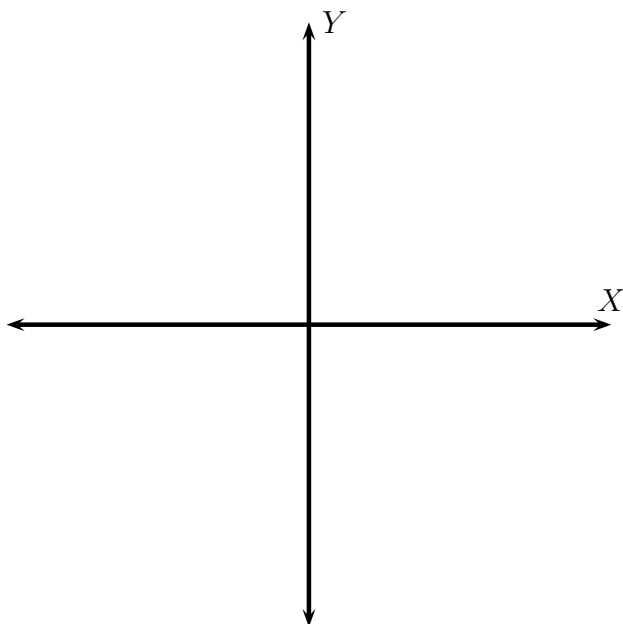


- (4) Draw the curve given by the equation $y = -(x - 4)^2 + 2$. What is the vertex? Does the curve open up or open down?

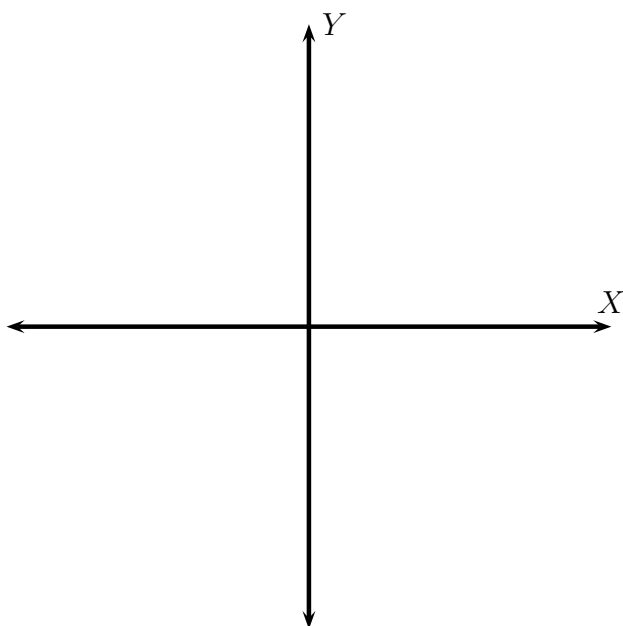


In general, the curve given by the equation $y = ax^2 + bx + c$ for $a \neq 0$ is a parabola with vertex whose x coordinate is given by $x = \frac{-b}{2a}$, and which opens up if $a > 0$, or opens down if $a < 0$.

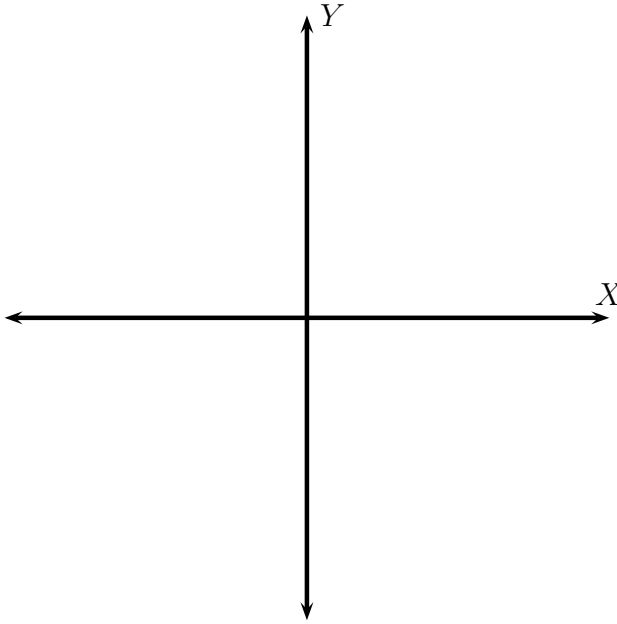
- (1) Draw the curve given by the equation $y = x^2 - 2x$. What is the vertex? Does the curve open up or open down?



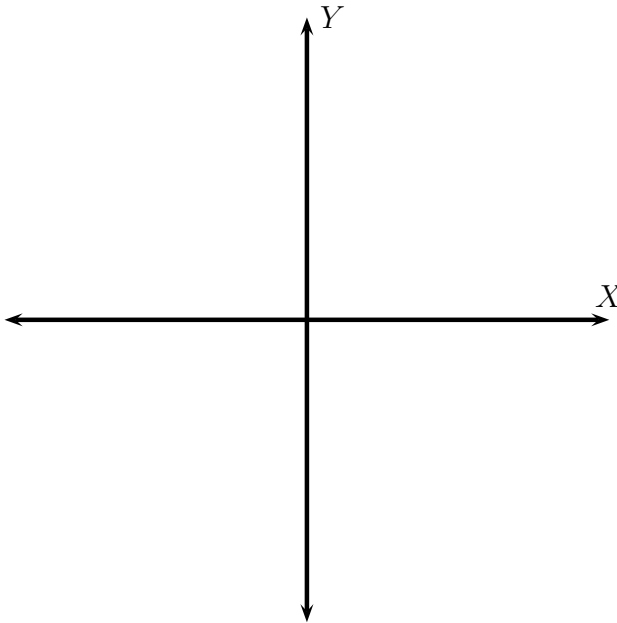
- (2) Draw the curve given by the equation $y = -x^2 + 2x + 2$. What is the vertex? Does the curve open up or open down?



- (3) Draw the curve given by the equation $y = -\frac{x^2}{3} - 2x - 2$. What is the vertex? Does the curve open up or open down?



- (4) Draw the curve given by the equation $y = -3x^2 + 12x - 11$. What is the vertex? Does the curve open up or open down?



Quadratic formula : The solutions to the quadratic equation $ax^2 + bx + c = 0$ is given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Solve for x :

(1) $x^2 - 8x - 16 = 4$.

(2) $-x^2 + 2x + 3 = -1$

End of review

(1) A projectile is launched into the air from the surface of planet A. On planet A, the height of any projectile y given in feet is determined by the equation $y = -6t^2 + v_0t$, where t is time in seconds and v_0 is the initial velocity of the object in feet per second. If the projectile is launched from the ground level with an initial velocity of 400 feet per second, then how many seconds will it take for the projectile to reach a height of 2506 feet?

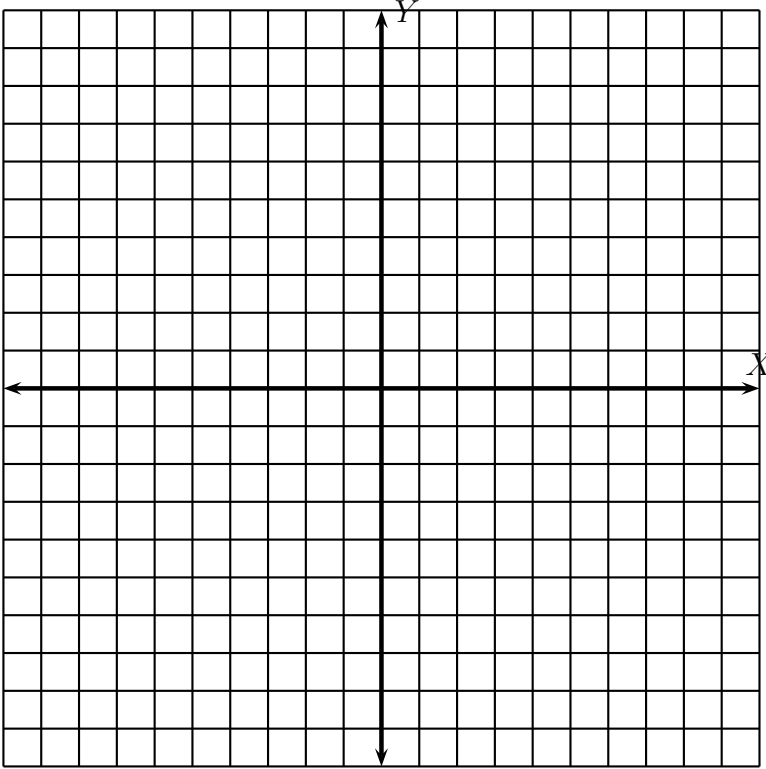
(2) At a local frog jumping contest, Rivet's jump can be approximated by the equation $y = -\frac{x^2}{4} + x$ and Croak's jump can be approximated by $y = -3x^2 + 6x$ where $x =$ length of the jump in feet and $y =$ height of the jump in feet.

- Which frog jumped higher? How high did it jump?

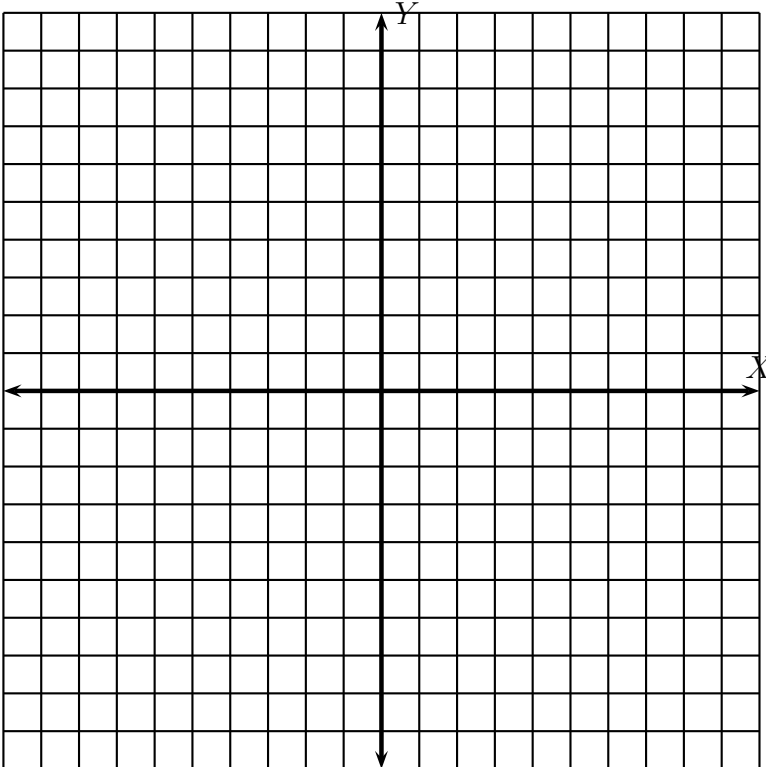
- Which frog jumped farther? How far did it jump?

LESSON 10.

- (1) Shade the region given by the inequality $-x + 3y \leq 9$.

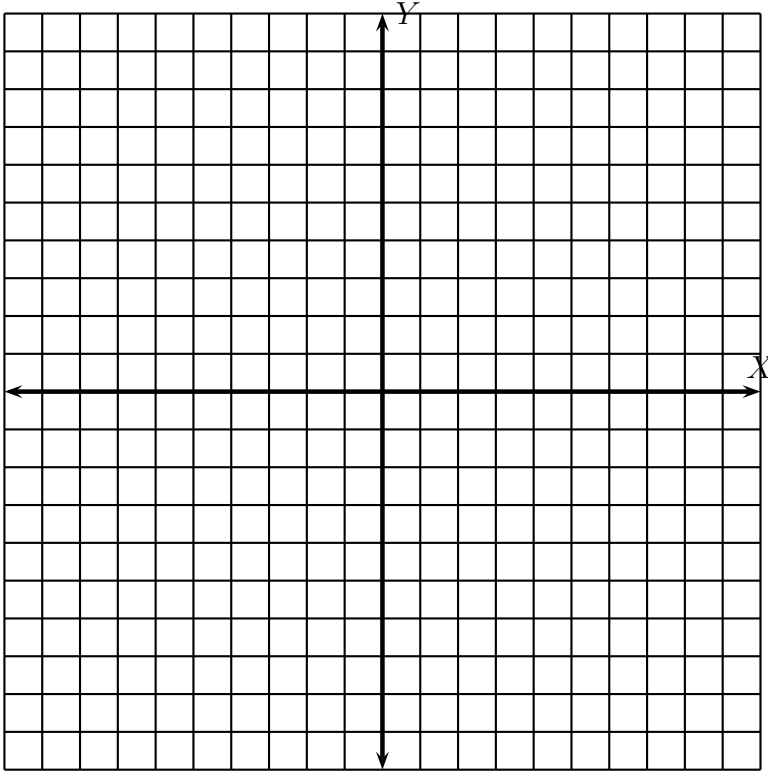


- (2) Shade the region given by the inequality $x + y \leq 7$.



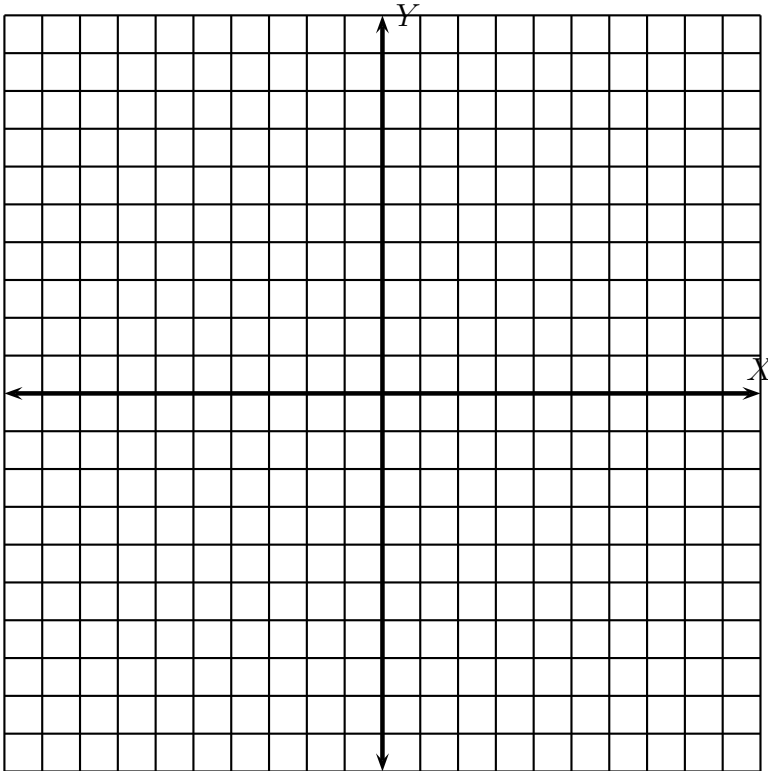
(3) Shade the region given by the inequalities:

$$-x + 3y \leq 9, \quad x + y \leq 7.$$



(4) Shade the region given by the inequalities:

$$-x + 3y \leq 9, \quad x + y \leq 7, \quad x \geq 0, \quad y \geq 0.$$



(5) What are the vertices or corner points in the previous problem?

The Fundamental Theorem of Linear Programming : The maximum or minimum value of the objective function will occur at a corner point of the feasible region.

(6) Maximize $P = 2x + 3y$ subject to the constraints

$$-x + 3y \leq 9, \quad x + y \leq 7, \quad x \geq 0, \quad y \geq 0.$$

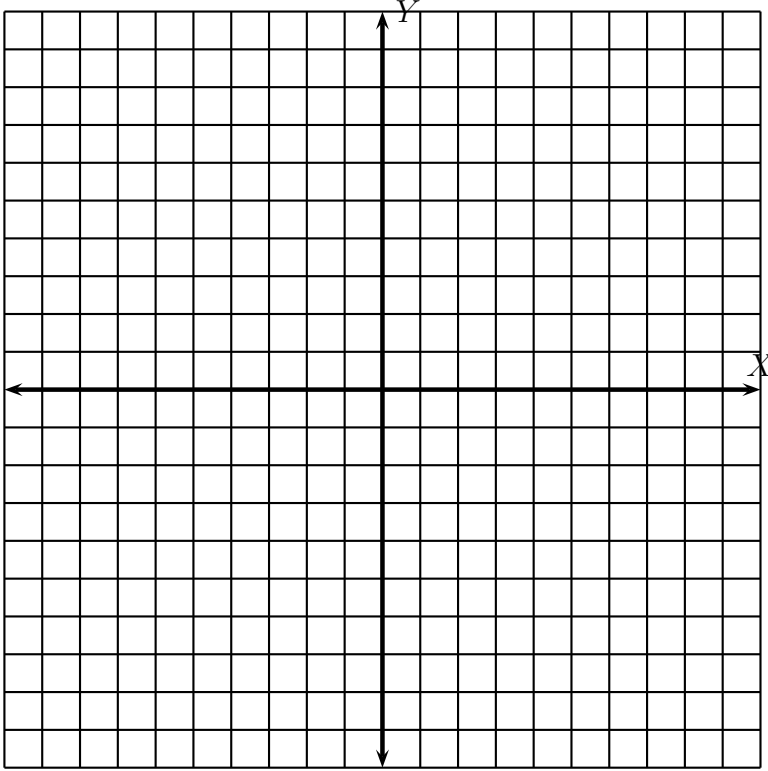
(7) Minimize $Q = 4x + 5y$ subject to the constraints

$$-x + 3y \leq 9, \quad x + y \leq 7, \quad x \geq 0, \quad y \geq 0.$$

This was the case of a **bounded** feasible region. In such cases, we will be able to find both the maximum and minimum for P and Q.

(8) Maximize $P = -5x + y$ subject to the constraints

$$3x + 2y \geq 5, \quad -x + 4y \leq 3, \quad x \geq 0, \quad y \geq 0.$$



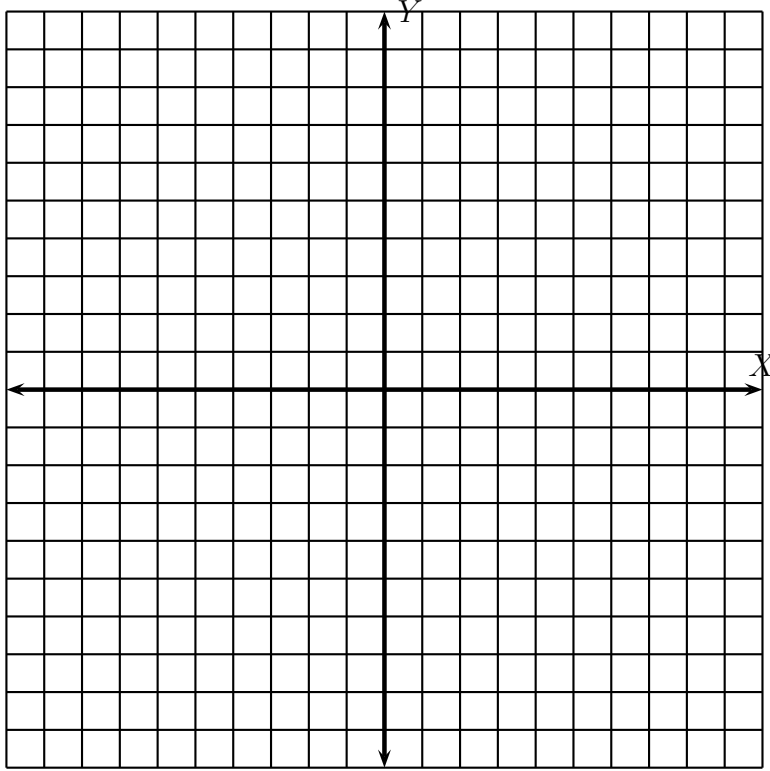
(9) Minimize $Q = 2x - 7y$ subject to the constraints

$$3x + 2y \geq 5, \quad -x + 4y \leq 3, \quad x \geq 0, \quad y \geq 0.$$

This was the case of an **unbounded** feasible region. In such cases, we will not be able to find minimum for P and maximum for Q.

(10) Maximize $P = x - 3y$ subject to the constraints

$$3x - y \leq 10, \quad 2x + 5y \geq 18, \quad x \geq 0, \quad y \geq 0.$$

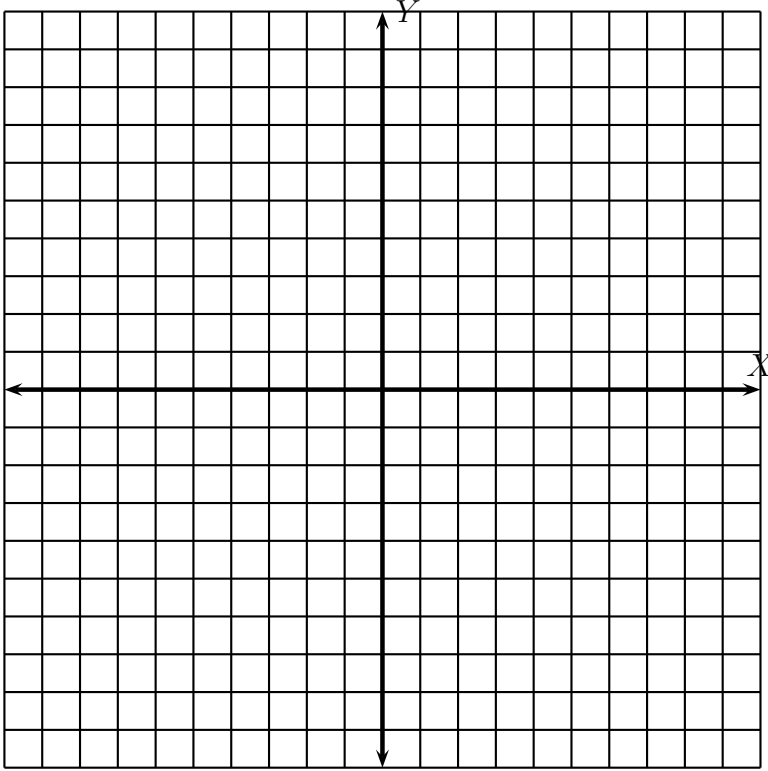


(11) Minimize $Q = 2x + y$ subject to the constraints

$$3x - y \leq 10, \quad 2x + 5y \geq 18, \quad x \geq 0, \quad y \geq 0.$$

(12) Maximize $P = 2x - 3y$ subject to the constraints

$$x + 10y \leq 50, \quad 4x - 7y \leq 12, \quad x \geq 0, \quad y \geq 0.$$



(13) Minimize $Q = x - 2y$ subject to the constraints

$$x + 10y \leq 50, \quad 4x - 7y \leq 12, \quad x \geq 0, \quad y \geq 0.$$

(14) Jay works no more than 20 hours per week. He is paid \$ 10 per hour for cutting lawns and \$ 7 per hour for baby-sitting. He wants to spend between 3 and 8 hours per week cutting lawns. Determine Jay's maximum earnings per week.

(15) A gardner wants to apply fertilizers to his garden. Type A fertilizer has 1% Nitrogen and 2% Phosphorus and costs \$ 5 per pound. Type B fertilizer has 2% Nitrogen and 1% Phosphorus and costs \$ 6 per pound. The soil needs at least 4 pounds of Nitrogen and 5 pounds of Phosphorus. Find the number of pounds of each fertilizer so as to have healthy soil at minimum cost.

LESSON 11.

The word **percent** means “out of a hundred.”

- (1) Change 35.49% to a decimal.
- (2) Change 99.99% to a decimal.
- (3) Change 44 out of 50 to a percent.
- (4) Change $\frac{2}{3}$ to a percent. (Round off to two places after decimal point).
- (5) Find 8.3% of 45. (Round off to two places after decimal point).
- (6) A bag costs \$ 85.52. If the sales tax is 7% , then how much would you actually pay for the bag?
- (7) A DVD-player costs \$ 123.00. There is a discount of 20%. What is the sale price of the player?
- (8) A bookstore has marked up 30 % the cost of all its books. What is the original cost of a book if it now costs
 - \$ 40?
 - \$ 90?

- \$ 120?

(9) The price of a book is \$ 72.

- The store offers a discount of 15% on the book. What is the new price of the book?

- You have a coupon which allows you a 25% discount on the final price of the book. How much do you pay for the book?

- Would a direct 40% discount on the book make it more or less expensive?

(10) If the price of a microwave increased from \$ 45 to \$ 60, then what was the percent increase in the price?

(11) If the price of a car dropped from \$ 22,000 to \$ 20,000, then what was the percent decrease in the price?

LESSON 12.

Simple interest formula :

P = Principal or amount deposited,

r = annual rate of interest,

t = time in years,

the **simple interest** I is given by the formula

$$I = Prt.$$

The **final amount** A is

$$A = P + I$$

$$A = P + Prt$$

$$A = P(1 + rt)$$

(1) Find the simple interest and final amount for:

- $P = \$2000$, $r = 4\%$ annually, $t = 8$ years.

- $P = \$2000$, $r = 4\%$ annually, $t = 8$ months.

- $P = \$2000$, $r = 4\%$ monthly, $t = 8$ years.

- $P = \$3000$, $r = 6\%$ annually, $t = 5$ months.

(2) Doris deposited \$ 25,000 in a bank offering simple interest eighteen months ago. If the amount in the bank currently is \$ 28,000 then what is the rate of interest the bank offered her?

(3) Ben deposited some amount two years ago at a rate of interest 1.5% per month. If the present amount is \$ 30,000 then what was the amount deposited?

LESSON 13.

The formula given in this handout is slightly different from what is given in the text although, the results are the same. The formula given here is more commonly used by authors, and is less confusing while dealing with the formula of **effective rate** later in the section.

Compound interest formula :

P = Principal or amount deposited,

r = rate of interest given annually (called, the **nominal interest rate**),

n = number of times the interest is compounded per year,

t = number of years,

A = the **final amount**.

$$A = P \left(1 + \frac{r}{n} \right)^{nt} .$$

The **compound interest** earned is

$$I = A - P$$

(1) Find the interest earned if

- $P = \$3,000$, $r = 4\%$ annually, $t = 3$ years, and the interest is compounded once a year.

- $P = \$3,000$, $r = 4\%$ annually, $t = 3$ years, and the interest is compounded semi-annually.

- $P = \$3,000$, $r = 4\%$ annually, $t = 3$ years, and the interest is compounded quarterly.

- $P = \$3,000$, $r = 4\%$ annually, $t = 3$ years, and the interest is compounded monthly.

- $P = \$3,000$, $r = 4\%$ annually, $t = 3$ years, and the interest is compounded daily.

A short review on solving equations: Solve for x

(1) $x^5 = 200$.

(2) $5^x = 200$.

(3) $x^8 = 5000$.

(4) $8^x = 5000$.

End of review. Now we return to the study of compound interest.

- (1) What was the amount deposited 8 years ago at the rate of interest of 5% compounded monthly, if the final amount A is \$ 6,000?
- (2) How long did it take for the principal of \$ 8,000 to grow to \$ 12,000, when the rate of interest at the time of deposit of was 7% compounded monthly?
- (3) Find the rate of interest compounded monthly, if the principal of \$10,000 grew to the final amount of \$ 15,000 after 7 years.

The **effective interest rate** is the simple interest rate, which over a one year period provides the same amount of interest as the nominal rate compounded gives. If r_{eff} denotes the effective interest rate, then the formula is given by

$$r_{eff} = \left(1 + \frac{r}{n}\right)^n - 1$$

where r denotes the nominal interest rate, and n is the number of times the nominal interest rate is compounded per year.

(1) Determine the effective interest rate if

- the nominal interest rate is 8% compounded once a year.

- the nominal interest rate is 8% compounded semi-annually.

- the nominal interest rate is 8% compounded monthly.

- the nominal interest rate is 8% compounded daily.

LESSON 14.

The Annuity Formula : The formula we give in this handout is different from the one given in the textbook, although the results are the same. The formula given here is consistent with the formula for compound interest.

Suppose PMT = the payment deposited on a regular basis,

r = the annual rate of interest,

n = number of deposits made per year,

the **final amount of annuity**, S is given by the formula

$$S = PMT \left[\frac{\left(1 + \frac{r}{n}\right)^{nt} - 1}{\frac{r}{n}} \right].$$

- (1) Suppose $PMT = \$100$ is deposited monthly for 5 years and the annual rate of interest $r = 6\%$ compounded monthly is offered, then find the final amount of annuity. What is the interest earned?

- (2) Suppose $PMT = \$200$ is deposited monthly for 5 years and the annual rate of interest $r = 6\%$ compounded monthly is offered, then find the final amount of annuity. What is the interest earned?

- (3) Suppose $PMT = \$100$ is deposited monthly for 5 years and the annual rate of interest $r = 7\%$ compounded monthly is offered, then find the final amount of annuity. What is the interest earned?

- (4) Suppose $PMT = \$100$ is deposited monthly for 10 years and the annual rate of interest $r = 6\%$ compounded monthly is offered, then find the final amount of annuity. What is the interest earned?

This is the end of our course. Best wishes.