Bronx Community College of City University of New York
Department of Mathematics and Computer Science

MATH 15 REVIEW SHEET

1. Integrate

(a) $\int 2^{3x} dx$ (b) $\int_1^2 x e^{3x^2} dx$ (c) $\int \ln x dx$ (d) $\int x^2 e^x dx$ (e) $\int \tan(4-5x) dx$

(f) $\int_0^{\pi} \sin^2 x dx$ (g) $\int \frac{dx}{3\sqrt{x}}$ (h) $\int 5t\sqrt{1-2t^2} dt$ (i) $\int \frac{2x}{x^2-1} dx$ (j) $\int \frac{2x^2+2x+1}{x^2} dx$

(k) $\int \frac{\sqrt{9-x^2}}{x^4} dx$ (l) $\int \frac{dx}{\sqrt{x^2+2}}$ (m) $\int_1^e \frac{11}{x} dx$ (n) $\int \frac{\ln x^2}{x} dx$ (o) $\int_{-3}^{3} \frac{\cos x}{x} dx$

2. Find the both average and the root-mean-square value of $f(x) = x^2$ in the interval [3,6].

3. A point start with initial speed $v_x = 3t, v_y = -2$ at the point (2,3) and moves along a curved path with x and y acceleration of $a_x = \cos t$ and $a_y = 4t^2 - 5$. Write the expression for the x and y components of displacement and what is the displacement when $t = 2$.

4. For the curves in (a) and (b), answer the following: (i) Find the volume generated by rotating the first-quadrant area bounded by the curves about the x-axis; (ii) Find the centroid of volume of revolution about the x-axis; (iii) Let a water tank be the volume generated as in (i), suppose the tank is full, how much work is needed to pump the water to a height of 5 units above the tank.

(a) $y = x^{5/2}$, $x = 1$ and $x = 2$ (b) $y = \sin x$, $x = 0$ and $x = 3\pi/4$

5. Find the area between the curves $y = x^4 - x^2$ and $y = 2$.

6. For the following curves, answer the following: (i) Find the intersection points of the curves; (ii) Find the area bounded by the intersection points of the curves; (iii) Find the centroid of the region bounded by the curves; (iv) Find the volume generated by rotating the area bounded by the curves about the y-axis.

(a) $y^2 = x$ and $y = x^3$ (b) $y = \cos x, y = \sin x$ for $x \in [0, 2\pi]$.

7. Find the area bounded by $y = x\sqrt{1+x}$ from $x = 1$ to 7 using (a) midoint method; (b) trapezoid method; (c) Simpson's Rule, for each above method, use 6 panels of equal length. (d) Find the exactvalue of area using integration.

8. Find the general solution to each differential equation.

(a) $y' = x^2 y^5$ (b) $\sqrt{1+x^2}dy + xydx = 0$ (c) $y' + 2xy = x$

(d) $y'' = \cos 2x$ (e) $5y'' + 7y' - 6y = 0$ (f) $y'' - 2y' + 2y = 0$

9. Find the particular solution to each differential equation.

(a) $xy' = 4x^4 - y$ given that $y = 2$ when $x = 1$.
(b) $y' \sin y = \cos x$ given that $x = \pi/6$ when $y = 0$.
(c) $y'' + 3y' + 2y = 0$ given that $y = 1$ and $y' = 2$ when $x = 0$.

10. Use the ratio test to determine, if possible, if each series converges or diverges.

(a) $1 - \frac{1}{2!} + \frac{1}{4!} - \frac{1}{6!} + \cdots + \frac{(-1)^n}{(2n)!} + \cdots$

(b) $1 + \frac{2^2}{2!} + \frac{3^3}{3!} + \frac{4^4}{4!} + \cdots + \frac{n^n}{n!} + \cdots$

(c) $1 + \frac{4}{7} + \frac{9}{49} + \cdots + \frac{n^2}{7^{n-1}} + \cdots$

11. Write four terms of a Taylor series for each function expanded about $a = 1$.

(a) $f(x) = e^{x^2}$

(b) $f(x) = \sqrt{x}$

Answer

1.

(a) $\dfrac{8^x}{\ln 8} + C$ (b) $\dfrac{e^{12} - e^3}{6}$ (c) $x \ln|x| - x + C$ (d) $e^x(x^2 - 2x + 2) + C$

(e) $\dfrac{\ln|\cos(4-5x)|}{5} + C$ (f) $\dfrac{\pi}{2}$ (g) $\dfrac{2\sqrt{x}}{3} + C$ (h) $\dfrac{-5(1-2t^2)^{3/2}}{6} + C$

(i) $\ln|x^2 - 1| + C$ (j) $2x + 2\ln|x| - \dfrac{1}{x} + C$ (k) $(\dfrac{1}{27x} - \dfrac{1}{3x^3})\sqrt{9 - x^2} + C$ (l) $\ln|\sqrt{x^2+2} + x| + C$

(m) 11 (n) $(\ln|x|)^2 + C$ (o) 0

2. The average $= 21$ and the root-square-mean $= \dfrac{9\sqrt{155}}{\sqrt{5}}$

3. $d_x = -\cos t + \dfrac{3}{2}t^2 + 3$ and $d_y = \dfrac{1}{3}t^4 - \dfrac{5}{2}t^2 - 2t + 3$; $d_x(2) = 9 - \cos 2$ and $d_y(2) = -5\dfrac{2}{3}$.

4.

(a) (i) $\dfrac{21\pi}{2}$ (ii) $(\dfrac{254}{147}, 0)$ (iii) $\dfrac{2,325\pi}{42}$

(b) (i) $\dfrac{3\pi^2 + 2\pi}{8}$ (ii) $(\dfrac{9\pi^2 + 12\pi + 8}{8(3\pi+2)}, 0)$ (iii) $\dfrac{\pi(9\pi^2 + 120\pi + 72)}{64}$

5. $\dfrac{56\sqrt{2}}{15}$.

6.

(a) (i) (1,1) (ii) $\dfrac{5}{12}$ (iii) $(\dfrac{12}{25}, \dfrac{3}{7})$ (iv) $\dfrac{2\pi}{5}$

(b) (i) $(\dfrac{\pi}{4}, \dfrac{\sqrt{2}}{2})$ & $(\dfrac{5\pi}{4}, \dfrac{-\sqrt{2}}{2})$ (ii) $\sqrt{2}$ (iii)$(1 + \dfrac{\pi}{4}, \dfrac{\sqrt{2}}{2})$ (iv) $\dfrac{\sqrt{2}\pi^2}{2} + 2\sqrt{2}\pi$

7. (a) 56.85012 (b) 57.13693 (c) 56.94654 (d) $\dfrac{604\sqrt{2}}{15} \approx 56.945666$

8.

(a) $4x^3 + \dfrac{3}{y^4} = C$ (b) $y = Ce^{-\sqrt{1+x^2}}$ (c) $y = \dfrac{1}{2} + Ce^{-x^2}$

(d) $y = \dfrac{-\cos 2x}{4} + C_1 x + C_2$ (e) $y = C_1 e^{0.6x} + C_2 e^{-2x}$ (f) $y = e^x(C_1 \cos 2x + C_2 \sin 2x)$

9. (a) $y = \dfrac{4x^4}{5} + \dfrac{6}{5x}$ (b) $2\cos y = -2\sin x + 3$ (c) $y = 4e^{-x} - 3e^{-2x}$

10. (a) converges (b) diverges (c) converges

11. (a) $e(1 + 2(x-1) + 3(x-1)^2 + \frac{10(}{3}(x-1)^3)$ (b) $1 + \frac{x-1}{2} - \frac{(x-1)^2}{8} + \frac{(x-1)^3}{16}$